UNIVERSITY OF ILLINOIS
URBANA

AERONOMY REPORT NO. 36

MEASUREMENT OF AVERAGE ELECTRON DENSITY BETWEEN 75 AND 80 KILOMETERS

by

D. A. Reynolds

C. F. Sechrist, Jr.

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Aeronomy Laboratory
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Supported by
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Grant ~~NGR-013~~ NGR-14-005-013

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ABSTRACT

Instrumentation capable of recording variables necessary for the computation of average electron density between 75 and 80 km has been developed. The instrumentation is used in conjunction with the existing partial-reflection facility at the University of Illinois. Data output of the instrumentation is recorded on paper tape in binary coded decimal form. To handle the paper tape, a program has also been developed which produces for each data acquisition interval a mean electron density at 77.5 km and data necessary for a time plot of electron density at 77.5 km.

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1. INTRODUCTION

The region of the ionosphere extending from 50 to 90 km is referred to as the D region. This region is composed in part of free electrons. To measure electron density as a function of height in this region radio frequency pulses are transmitted vertically from a ground-based station into the ionosphere. As the radio wave travels into the D region, discontinuities in the D region are encountered, and the radio wave is partially reflected. These reflections are detected at the ground-based station in the form of voltages produced by a receiving system due to the incidence of the reflected waves on the receiver system's antenna. From the knowledge of the magnitude of these voltages and from a knowledge of various atmospheric parameters one is able to ascertain electron density as a function of height.

One of the many questions which remains to be answered about the D region is the question of geographical variability of electron density. Data available in this area have been obtained from both rocket-borne and ground-based experiments. But, researchers employed in this general area have worked essentially independently of each other, and no coordinated study has been made. However, such a study would be necessary to answer many of the questions concerning geographical variability of electron density in this region. To make such a study possible a North American network of D region sounders was proposed. The name suggested for this network was SOMED (Synoptic Observation of Mesospheric Electron Densities). The proposed network will be composed, initially, of five sites; Saskatoon, Canada; Boulder, Colorado; Ottawa, Canada Urbana, Illinois; and State College, Pennsylvania. The last station, State College, Pennsylvania, utilizes the wave interaction method of measuring

electron densities instead of the partial-reflection method. The function of this network will be the coordinated measurement at the sites involved of average electron density around 75 km. Measurements will be taken every Wednesday at a solar zenith angle of 75 degrees at every site. Described in this report is the partial-reflection facility at Urbana, Illinois, and, in particular, the portion of the facility developed for use as a part of the SOMED network. It is the development of this latter portion and the data processing technique associated with it which comprise the problem of this report.

2. THE METHOD AND BASIC THEORY

The partial-reflection experiment uses the principle of differential absorption to determine electron density in the D-region. This principle is explained as follows. Consider a radio wave, polarized in the extraordinary mode of circular polarization, propagating in a magneto-ionic medium. Over a given distance this wave suffers an absorption which is greater than that which a wave of the ordinary mode of circular polarization would suffer. This difference in energy absorbed between the extraordinary mode and the ordinary mode of circular polarization is termed differential absorption. This differential absorption is proportional to the electron density in the region through which the radio waves propagates, and it is this differential absorption which is indirectly measured by the partial-reflection experiment and used to deduce electron density.

To consider the problem in more specific terms suppose a transmitter located on the ground transmits vertically toward the ionosphere, the ionized portion of the atmosphere, pulses of the ordinary and then the extraordinary mode in succession with only a short time interval between the pulses. See Figure 2.1. Now suppose the ordinary mode is transmitted first. As this wave moves into the ionosphere it suffers attenuation. Also, any sharp discontinuity in ionization will reflect a portion of that wave. Suppose the first discontinuity is encountered at $h = h_1$. If Fresnel reflection from a boundary between two media is assumed for the reflection at the discontinuity, the reflection coefficient is given (Budden, 1961) by

$$R = \frac{n_2 - n_1}{n_2 + n_1} \sim \frac{\delta n}{2n} \tag{2.1}$$

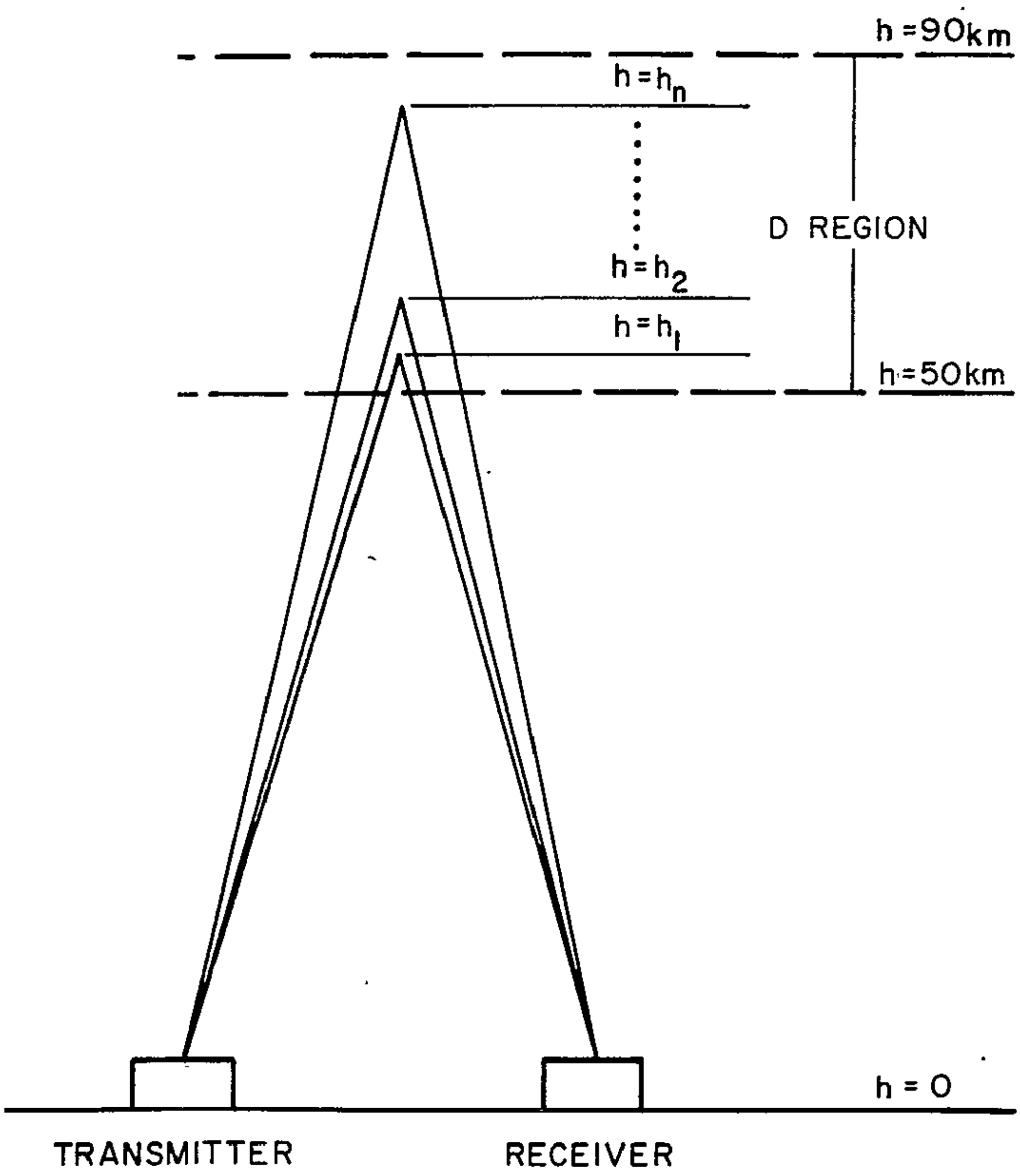


Figure 2.1 The partial-reflection model.

ω = angular frequency of the transmitted wave,

ν_m = monoenergetic collision frequency.

$$\mathcal{C}_p(x) = \frac{1}{p!}\int_0^\infty \frac{\varepsilon^p}{\varepsilon^2+x^2}\, e^{-\varepsilon} d\varepsilon$$

where

ω_L = gyro-frequency of the electron

$\varepsilon = mv^2/2kT$

ω_o^2 = plasma frequency = $Ne^2/m\varepsilon_o$

m = electron mass = 9.1×10^{-31}kg

v = electron velocity

k = Boltzmann constant = 1.38×10^{-23} J°K^{-1}

T = temperature

N = electron density

e = electron charge = 1.6×10^{-19}C

ε_o = permittivity of free space = 8.85×10^{-12}F m^{-1}

However, the expression for n^2 can be greatly simplified if the angle between the direction of propagation and the earth's magnetic field is sufficiently small. At Urbana, Illinois, this approximation (called the quasilongitudinal approximation) can be made (Pirnat, 1968) and the result is

$$n^2 = 1-[\omega_o^2(\omega\pm\omega_L)/\omega\nu_m^2]\, \mathcal{C}_{3/2}[(\omega\pm\omega_L)/\nu_m] -i(5\omega_o^2/2\omega\nu_m)\, \mathcal{C}_{5/2}[(\omega\pm\omega_L)/\nu_m] \tag{2.3}$$

For the ordinary polarization the sign associated with the constant ω_L is positive. (For extraordinary polarization the sign is negative.) Equation 2.3 leads (Pirnat, 1968) to an ordinary model reflection coefficient given by

where n_1 is the refractive index of the lower medium, n_2 is that of the upper medium, and n an average between the two. For the ionosphere (Sen and Wyller, 1960) n is given by

$$n^2 = (A + B\sin^2\phi \pm \sqrt{B^2\sin^4\phi - C^2\cos^2\phi})/(D + E\sin^2\phi) \qquad (2.2)$$

where

$$A = 2\varepsilon_I(\varepsilon_I+\varepsilon_{III})$$

$$B = \varepsilon_{III}(\varepsilon_I+\varepsilon_{III}) + \varepsilon_{II}{}^2$$

$$C = 2\varepsilon_I\varepsilon_{II}$$

$$D = 2\varepsilon_I$$

$$E = 2\varepsilon_{III}$$

$$\varepsilon_I = (1-a) - ib$$

$$\varepsilon_{II} = (f-d)/2 + i(c-e)/2$$

$$\varepsilon_{III} = [a - (c+e)/2] + i[b - (f+d)/2]$$

$$a = (\omega_o^2/\nu_m^2)\mathcal{C}_{3/2}(\omega/\nu_m)$$

$$b = (5\omega_o^2/2\omega\nu_m)\mathcal{C}_{5/2}(\omega/\nu_m)$$

$$c = (\omega_o^2(\omega-\omega_L)/\omega\nu_m^2)\mathcal{C}_{3/2}[(\omega-\omega_L)/\nu_m]$$

$$d = (5\omega_o^2/2\omega\nu_m)\mathcal{C}_{5/2}[(\omega-\omega_L)/\nu_m]$$

$$e = [\omega_o^2(\omega+\omega_L)/\omega\nu_m^2]\mathcal{C}_{3/2}[\omega+\omega_L)/\nu_m]$$

$$f = (5\omega_o^2/2\omega\nu_m)\mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m]$$

The definitions of the quantities presented in the above equations are

ϕ = angle between the direction of propagation and the earth's magnetic field,

$$R_o = -\ \delta N(e^2/m\varepsilon_o\omega\nu_m)\{[(\omega+\omega_L)/\nu_m]\mathcal{C}_{3/2}[(\omega+\omega_L)/\nu_m] \\ +i(5/2)\mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m]\} \tag{2.4}$$

The reflected wave suffers attenuation on the return path just as it did before reflection; and after passing out of the ionosphere it is eventually incident upon the receiving antenna. A voltage is produced at the output of the receiver which is a function of the amplitude of the reflected wave.

If the time interval between pulses is short enough, the model of the D region can be assumed invariant for both transmitted modes. If this is so, both modes will be reflected by the same discontinuities. The extraordinary mode is therefore also reflected at $h = h_1$. Its reflection coefficient is given (Pirnat, 1968) by

$$R_x = -\delta N(e^2/m\varepsilon_o\omega\nu_m)\{[(\omega-\omega_L)/\nu_m]\mathcal{C}_{3/2}[(\omega-\omega_L)/\nu_m] \\ +i(5/2)\mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m]\} \tag{2.5}$$

However, the output voltage of the receiver will be smaller for the extraordinary mode due to the increased absorption suffered by this mode. The ratio of the two voltage amplitudes is a measure of the differential absorption

There is a minimum height below which no reflections are produced. This height is approximately 55 km. However, above this height discontinuities are present throughout the D region. As a result, reflection occurs not only at $h = h_1$, but throughout the entire D region. Thus, for a period of time after the occurrence of the transmitted pulses, no reflections are received and the receiver output is minimum. This delay can be calculated and for reflections occurring at an altitude no lower than 55 km the time delay is at least

$$t = \frac{2h}{c} = \frac{2(55\times10^3)}{3\times10^8}s = 367\ \mu s$$

(This value is an approximation since it assumes a mirror-like, or specular, reflection like that shown in Figure 2.1. In actuality there is not a mirror-like reflection but a refractive effect.) After this delay a series of peaks are received which correspond to reflections above 55 km, i.e., from the D region. Thus, the receiver output is a voltage vs. time profile similar to that shown in Figure 2.2. At t = 0, the transmitter transmits a pulse of the ordinary mode. For a time τ_1, no reflections are received. A series of peaks are then received which correspond to reflections from the D region. In terms of Figure 2.1, the peak at τ_1, would correspond to a reflection from h_1, τ_2 from h_2, and τ_n from h_n. Also note that τ_1 would therefore equal $\frac{2h_1}{c}$, $\tau_2 = \frac{2h_2}{c}$, and $\tau_n = \frac{2h_n}{c}$. At t = x, the extraordinary mode is transmitted, and once again there is a delay before any reflections are received. Since the model of the D region is assumed to be constant during the ordinary-extraordinary transmission cycle, the first reflection should be received at $t = x+\tau_1$, the second at $t = x+\tau_2$, and $\tau_n = x+\tau_n$. Also since the extraordinary wave suffers more attenuation the amplitude of each peak should be smaller than that of the corresponding peak for the ordinary transmission.

Since the D region of the earth's atmosphere is believed to be turbulent, it is difficult to decrease the interval between transmission of the ordinary and extraordinary mode low enough to consider the model of the D region to be invariant for the ordinary-extraordinary transmission cycle. As a result the peaks of the ordinary receiver voltage waveform and that of the extraordinary receiver voltage waveform do not always coincide in height. Also,

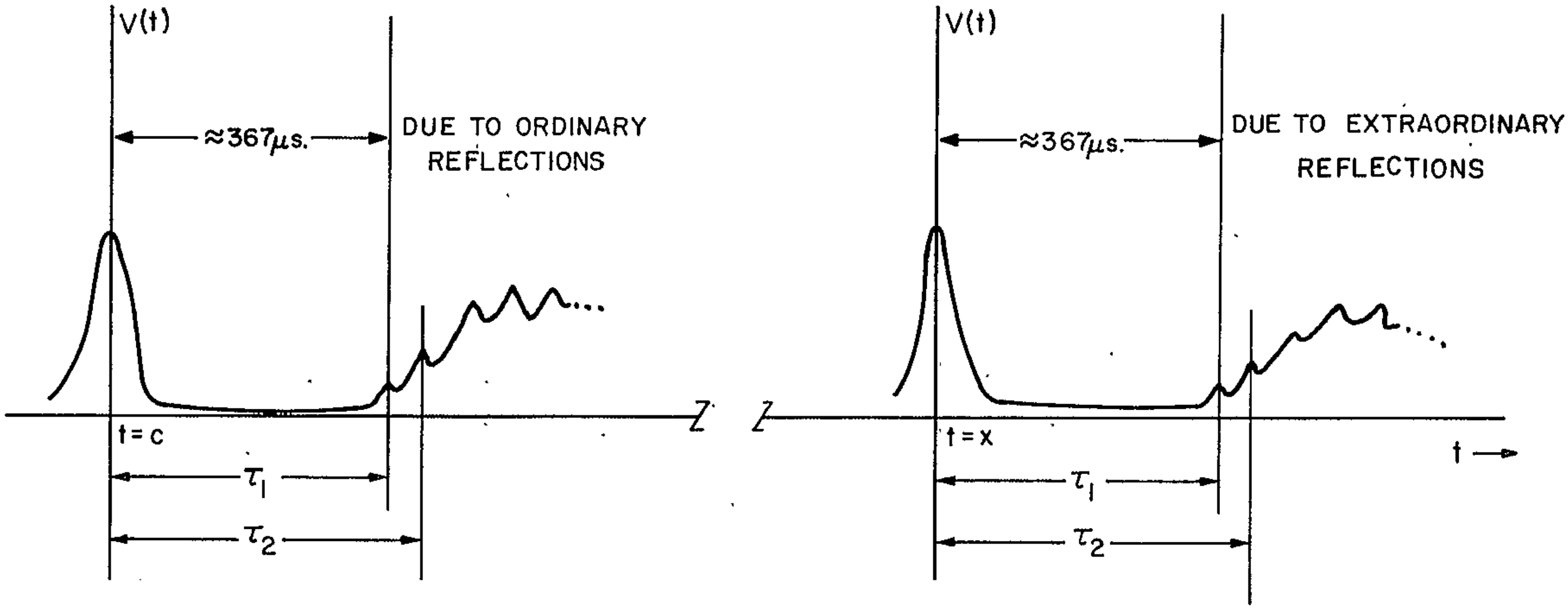


Figure 2.2 Receiver output voltage versus tim

even when peaks do coincide it is observed that the amplitude of the ordinary wave is sometimes smaller than that of the extraordinary wave; the short-term dynamics are possibly responsible for this. However, even under these restrictions the method has proven effective in determining D-region electron densiti

In order to determine electron density at a particular altitude h from a receiver voltage waveform like that shown in Figure 2.2, the ratio of extraordinary voltage amplitude to ordinary voltage amplitude must be determined at two heights in the vicinity of h, one at $(h + \frac{\Delta h}{2})$ and one at $(h - \frac{\Delta h}{2})$. These two values and the magnitude of the absolute value of the ratio of the extraordinary reflection coefficient to the ordinary reflection coefficient at both $(h + \frac{\Delta h}{2})$ and $(h - \frac{\Delta h}{2})$ are sufficient to determine electron density. The ratio of the coefficients at any altitude h (Pirnat, 1968) is given by

$$\left|\frac{R_x}{R_o}\right| = \left[\frac{\{[\omega-\omega_L)/\nu_m]\mathcal{C}_{3/2}[(\omega-\omega_L)/\nu_m]\}^2 + \{(5/2)\mathcal{C}_{5/2}[(\omega-\omega_L)/\nu_m]\}^2}{\{[\omega+\omega_L)/\nu_m]\mathcal{C}_{3/2}](\omega+\omega_L)/\nu_m]\}^2 + \{(5/2)\mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m]\}^2}\right]^{1/2} . \quad (2.6)$$

Hére the dependence on h is implicit in ν_m, the collision frequency. In order to calculate the reflection coefficient ratio a profile of collision frequency versus height must be assumed. Under this assumption a profile of $|Rx/Ro|$ is generated, and $|Rx/Ro|$ can be determined at $(h + \frac{\Delta h}{2})$ and $(h - \frac{\Delta h}{2})$. Electron density at h is therefore determined and is given (Pirnat, 1968) by

$$N = [(5\Delta he^2/2cm\varepsilon_o\nu_m)\{\mathcal{C}_{5/2}[(\omega-\omega_L)/\nu_m] - \mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m]\}]^{-1}$$

$$\times\ (\ln\{[(A_x/A_o)/(R_x/R_o)]_{h_1}/[(A_x/A_o)/(R_x/R_o)]_{h_2}\}) \quad (2.7)$$

where

ν_m = collision frequency at h,

$\frac{A_x}{A_o}$ = ratio of extraordinary receiver voltage amplitude to ordinary receiver voltage amplitude,

$\frac{R_x}{R_o}$ = absolute value of ratio of extraordinary reflection coefficient to ordinary reflection coefficient,

$h_1 = (h - \frac{\Delta h}{2})$,

$h_2 = (h + \frac{\Delta h}{2})$

In summary, from an assumed collision frequency profile a profile of $|Rx/Ro|$ is generated. From the receiver voltage vs. time profile a profile of Ax/Ao is generated. Using Equation (2.7), an electron-density profile is calculated.

To satisfy the requirements of the SOMED network, only the electron density around 75 km is required. At Urbana, Illinois, electron density will be computed at 77.5 km for the SOMED network, and the value used for Δh will be 5 km; thus $(h + \frac{\Delta h}{2})$ = 80 km and $(h - \frac{\Delta h}{2})$ = 75 km. Ax/Ao and Rx/Ro at 75 and 80 km are therefore required in order to determine N at 77.5 km. Equation (2.7) becomes

$$N(77.5\text{x}10^3) = [(5(5\text{x}10^3)e^2/2cm\varepsilon_o\nu_m(77.5\text{x}10^3))$$
$$\text{x}\{\mathcal{C}_{5/2}[(\omega-\omega_L)/\nu_m(77.5\text{x}10^3)] - \mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m(77.5\text{x}10^3)]\}]^{-1} \quad (2.8)$$
$$\text{x}(\ln\{[Ax/Ao)/(Rx/Ro)]_{75\text{x}10^3}/[(Ax/Ao)/(Rx/Ro)]_{80\text{x}10^3}\})$$

3. EXISTING PARTIAL-REFLECTION SYSTEM

The partial-reflection system at Urbana, Illinois, is composed of two antenna arrays (one each for transmitting and receiving), a 50 KW transmitter, and a sensitive receiver. A photograph of the electronic equipment is shown in Figure 3.1. The transmitter and receiver is described first

3.1 The Transmitter and Receiver

The parameters of the transmitter and the receiver are shown in tabular form in Table 3.1. The transmitter will be considered first. The duty cycle of the transmitter begins with the transmitting antenna phased for transmission of the ordinary mode of circular polarization; and energy supplied by the transmitter to the antenna will be radiated vertically in this mode. At t=0 the transmitter is triggered by a 10 μ s pulse. Responding to this trigger, the transmitter feeds the transmitting antenna array with a radio frequency pulse of 50 μs duration. Several milliseconds later a polarization control is actuated which changes the phasing of the transmitting antenna from ordinary to extraordinary polarization. Later, at t=33 ms. a second trigger is received by the transmitter, and the extraordinary pulse is transmitted. Several milliseconds later the polarization control is again actuated, and the antenna phasing is returned to the ordinary mode. At t=2 seconds the cycle is repeated. A block diagram of the transmitter is shown in Figure 3.2.

The receiver used in the partial-reflection experiment is of the superheterodyne type with all units shielded from external RF fields. A block diagram of this receiver is shown in Figure 3.3. The design is such that the maximum gain of the receiver is approximately 120 dB. The gain is controlled externally by varying the RF amplifier and IF amplifier gain. However,

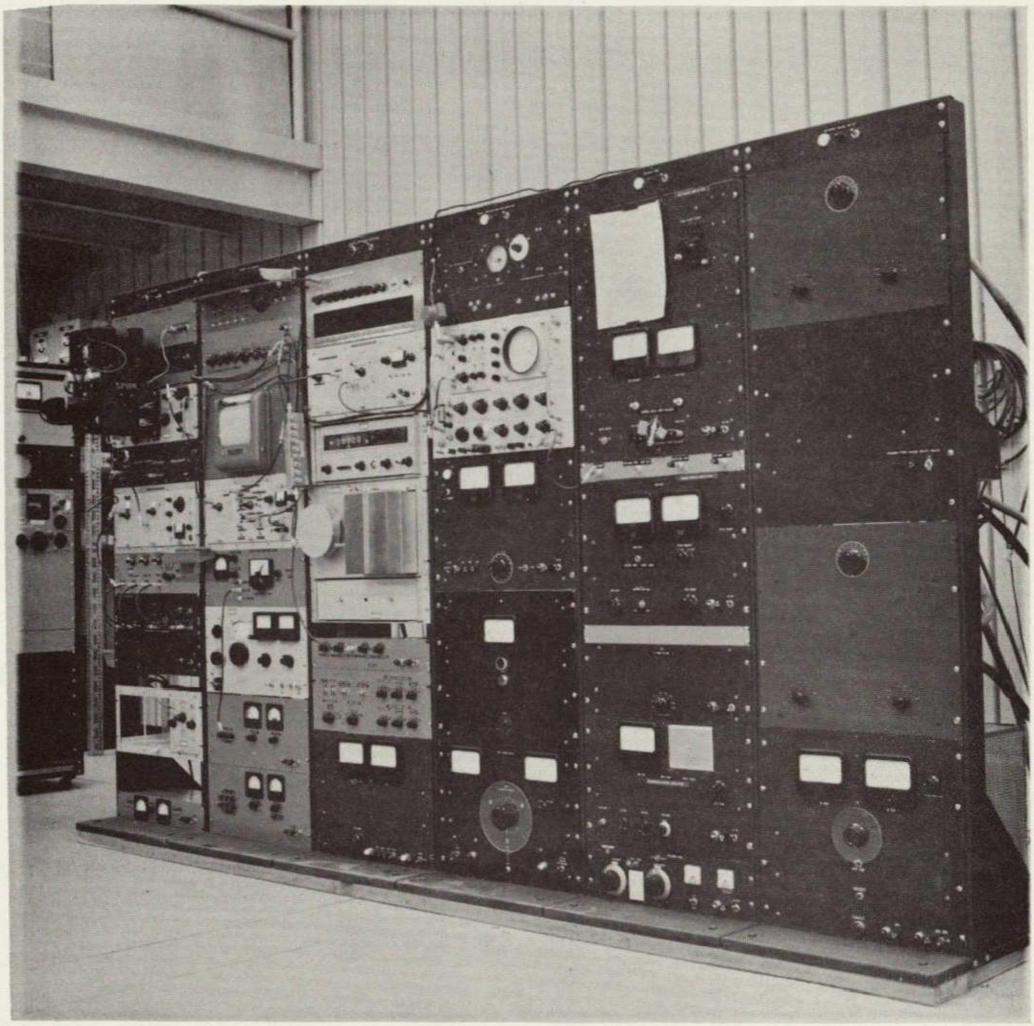


Figure 3.1 Partial reflection system.

TABLE 3.1

Transmitter and receiver parameters

TRANSMITTER

Frequency	2.66 MHz
Peak Power	50 KW
Pulse Width	50 μs
Pulse Repetition Rate	Double Pulse/2 sec
Output Impedance	50Ω, unbalanced

RECEIVER

Center Frequency	2.66 MHz
Noise Figure	3 dB Max
Bandwidth	35 KHz
Ripple Within Passband	3 dB Max
Recovery Time	200 μs after removal of 0.1 V_0 rms input
Gain Variation	3 dB Max, 15° to 35°C
RF Input Impedance	50Ω, unbalanced
Output Impedance	10 kΩ Max, unbalanced
Output Response	DC to 50 KHz, 10V_0 Max

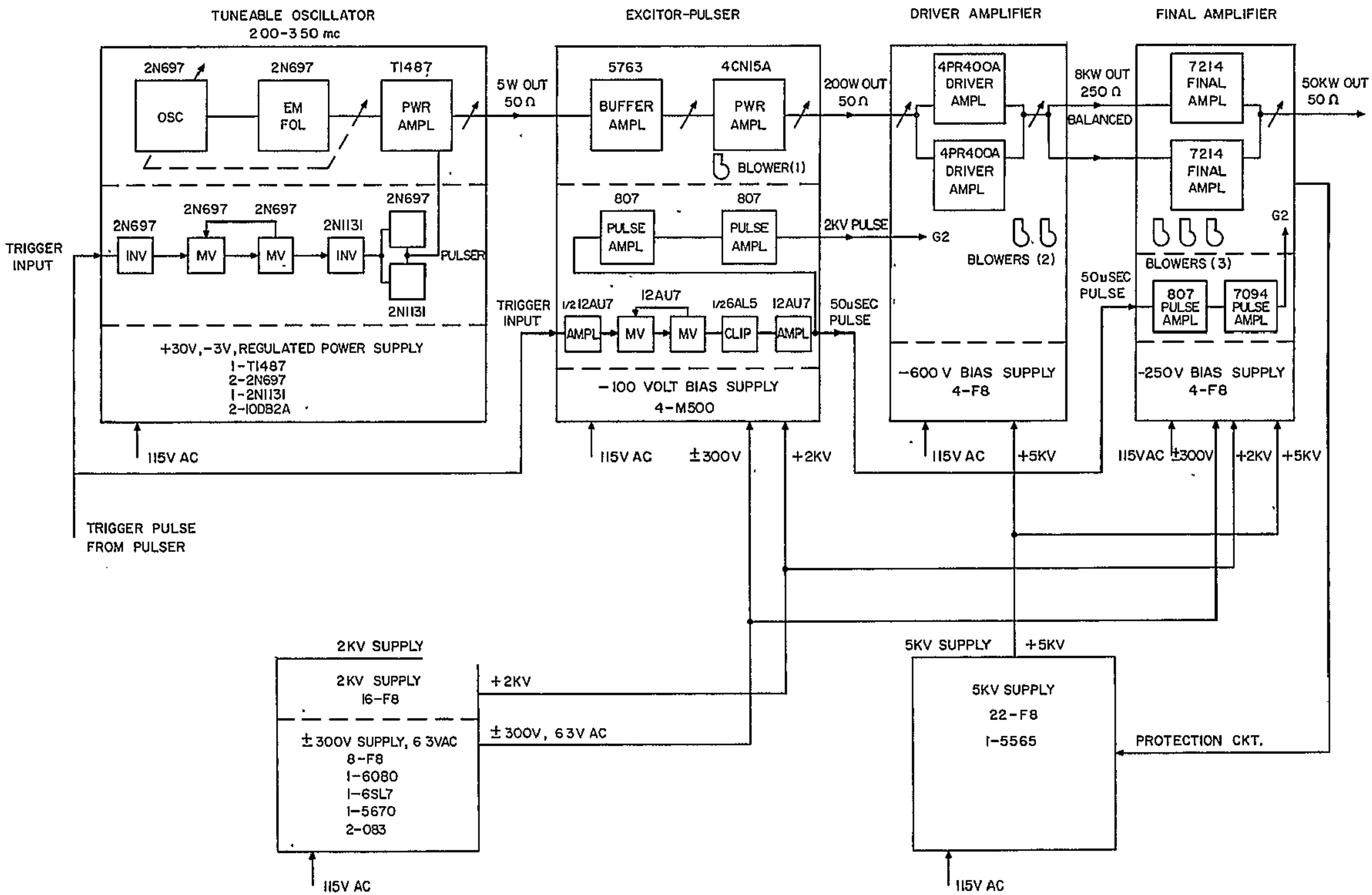


Figure 3.2 Block diagram of the transmitter (after Henry, 1966).

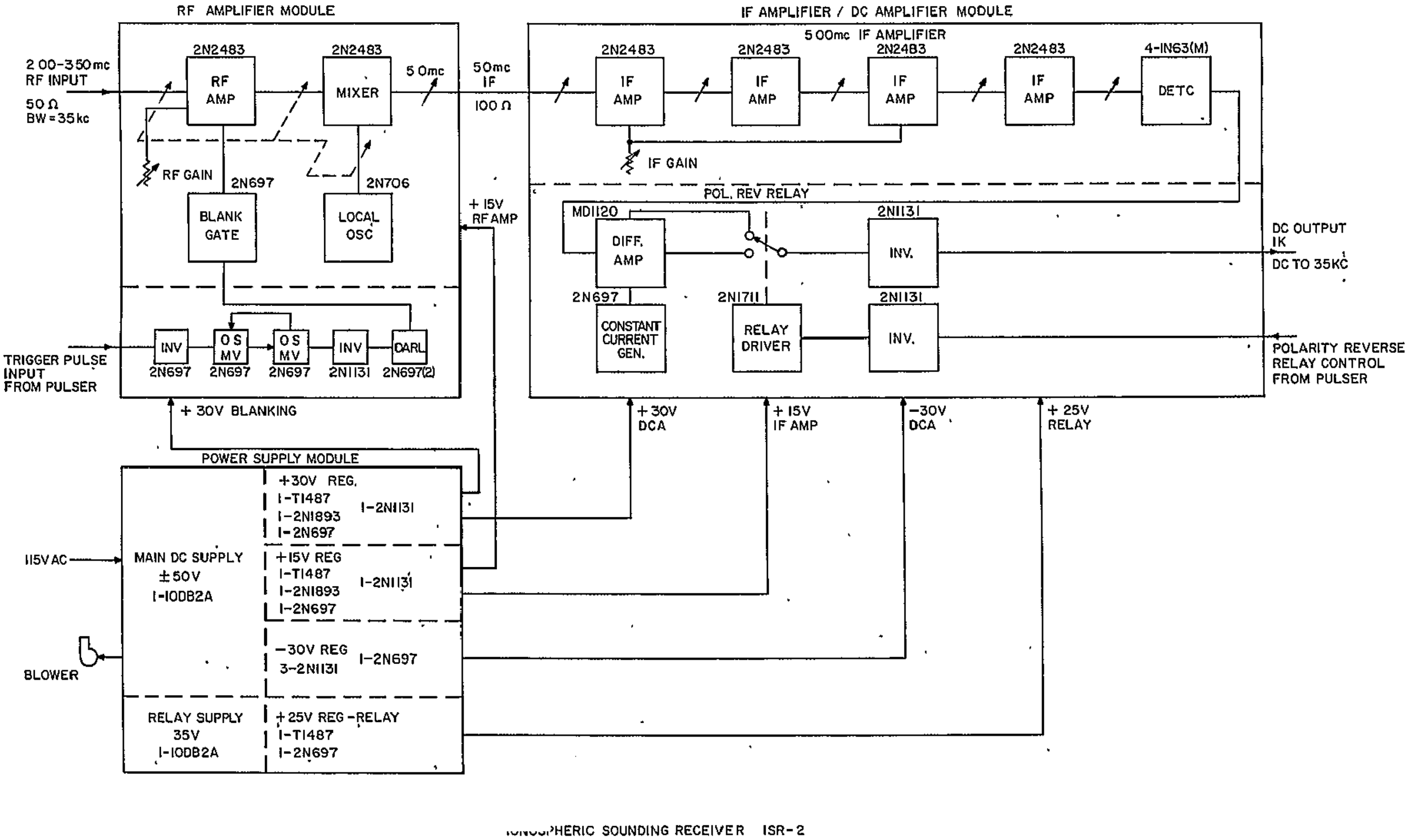


Figure 3.3 Block diagram of the receiver (after Henry, 1966).

in order to keep noise generated by the receiver to a minimum the RF amplifier gain is adjusted to a maximum, and the IF gain is set such that receiver output voltage is of the desired level. An attenuator pad is also used on the input line to the receiver as an additional RF input control. The detector of the receiver uses four germanimum diodes connected in a full-wave bridge circuit rather than the conventional half-wave detector system in order to reduce detector pulse distortion. Blanking circuitry is also designed into the receiver so as to protect the RF amplifier circuitry and to prevent complete receiver saturation. The latter is desirable since recovery time from complete receiver saturation is on the order of 200 μs, which is excessive since the first partial reflections are received approximately 350 μs after transmission.

In the discussion of the transmitter, mention was made of phasing the transmitting antenna in order to achieve ordinary and then extraordinary transmission. The polarization controls and the antenna system will be discussed next.

3.2 The Antenna System

The antenna system is composed of two arrays, one for transmitting and one for receiving. Their relative positions are as shown in Figure 3.4. Both arrays are identical and each consists of 60 half-wave dipoles. Consider a particular array, receiving or transmitting. Of the sixty half-wave dipoles in the array, thirty are aligned in a north-south direction, and thirty are aligned in an east-west direction. The thirty half-wave dipoles which are aligned in the north-south direction are grouped into 6 lines of 5 half-wave dipoles, and the 6 lines are spaced 1/2 wavelength apart and parallel.

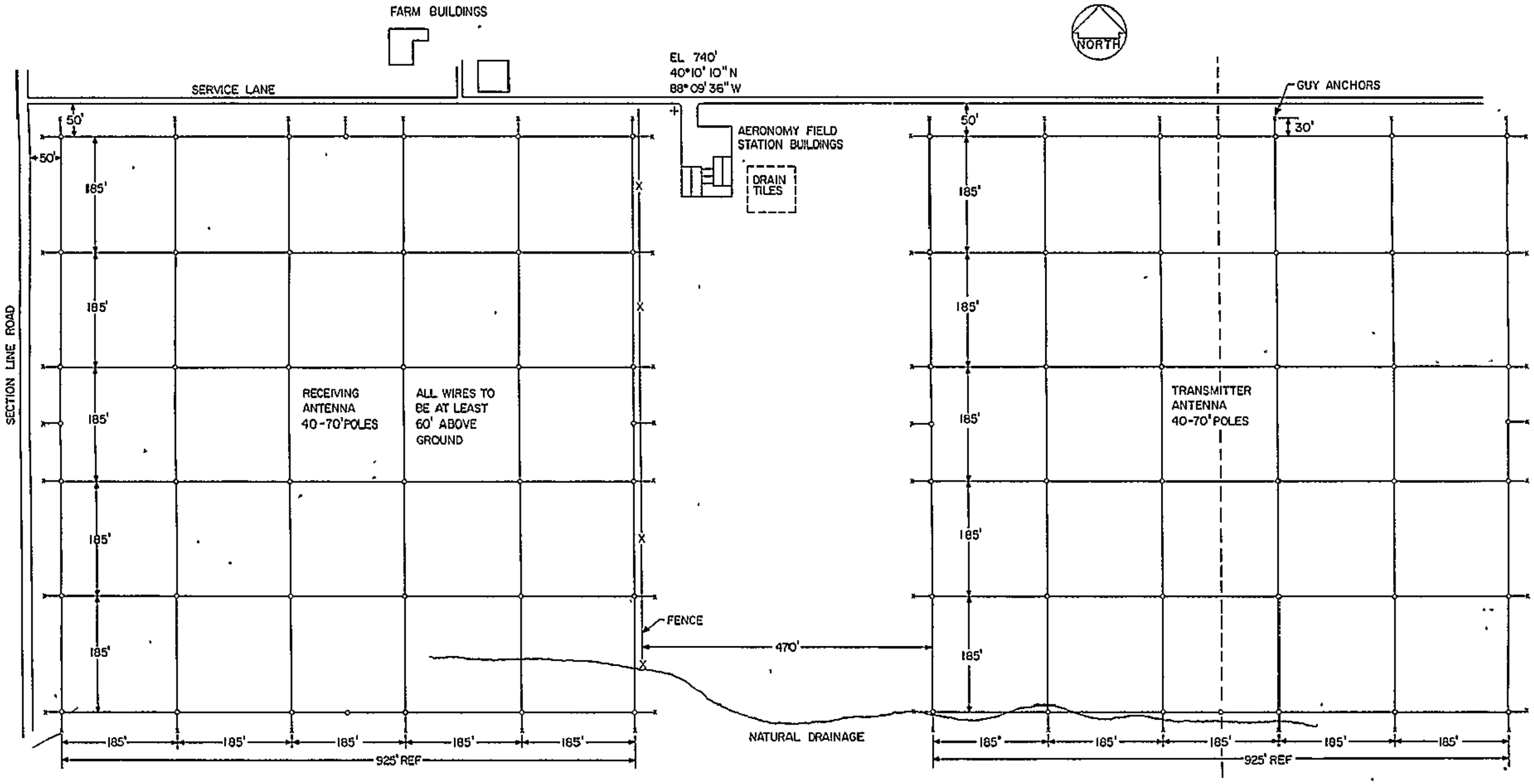


Figure 3.4 Antenna layout (after Henry, 1966).

A similar arrangement is used for the east-west direction, and the result is a square array for receiving and transmitting as shown in Figure 3.4.

In order to produce a circularly polarized transmitted wave the lines of the transmitting array which run in the east-west direction must be fed 90° out of phase with those in the north-south direction. Also, each must be fed an equal amount of power; otherwise the polarization will be elliptical. The system used to achieve these effects is shown in block diagram form in Figure 3.5. The power divider and attenuators are used to match the power supplied to each line. The 90° phase shift between lines is introduced as shown. Two matching networks are used (one in each line) for impedance transforming purposes between the 50-ohm feed line and the 600-ohm antenna system. Switching between the ordinary and the extraordinary mode of circular polarization is controlled by the polarization reversal control, and the actual transmission change is accomplished by introducing a 180° phase shift as shown. The reversal is, of course, made between the ordinary pulse and the extraordinary pulse transmission.

The receiver antenna system must be phased for reception of the reflection from the D region of the ordinary and extraordinary pulses transmitted by the transmitter. The phase switching of the receiver therefore occurs in unison with that of the transmitting antenna system. A block diagram of the system is shown in Figure 3.6. The polarization control is the same as that of the transmitter's phasing system, and the matching networks, attenuators, and phase shift networks perform similar functions in the receiver phasing system as they did in the transmitter phasing system. The receiver input is taken as the sum of the signals received on the 30 north-south dipoles and that received on the 30 east-west dipoles of the receiving system.

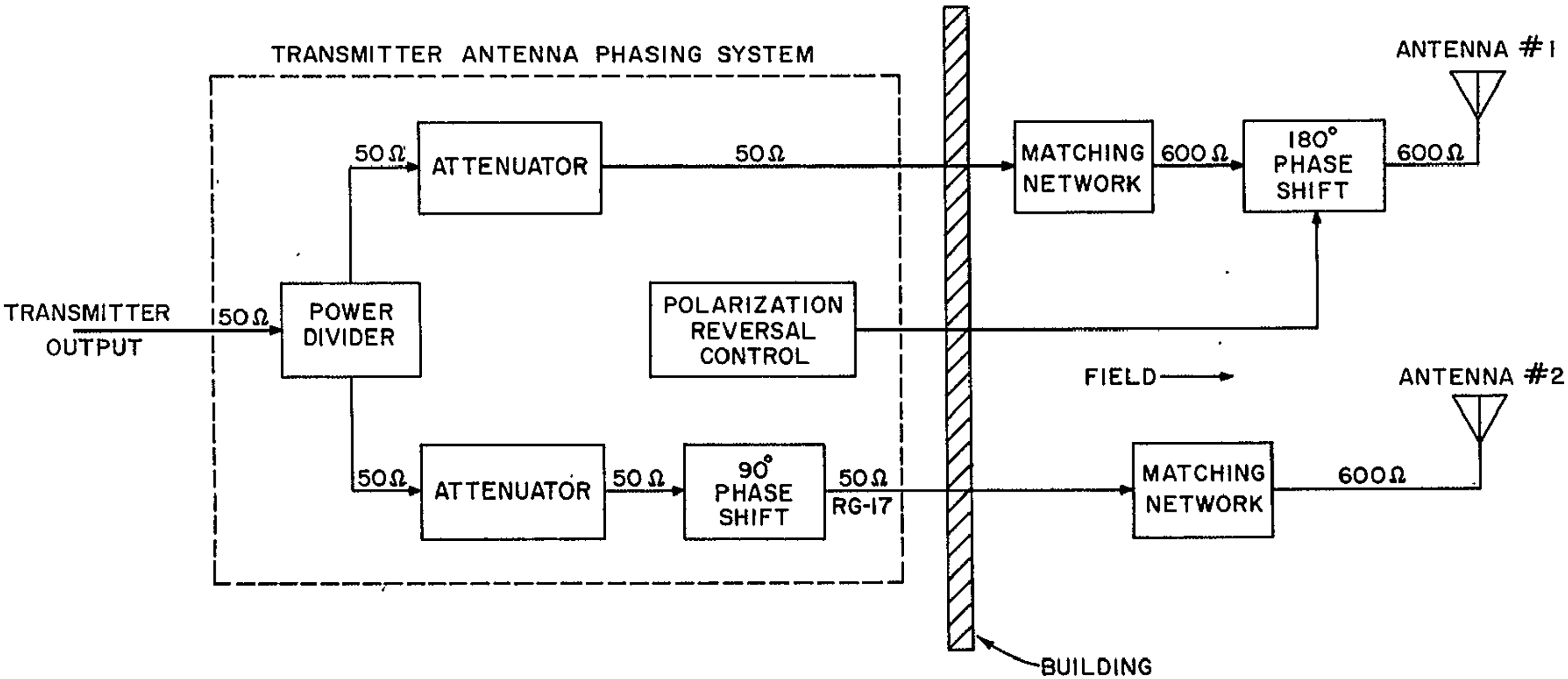


Figure 3.5 Block diagram of the transmitter antenna phasing system (after Pirnat, 1968).

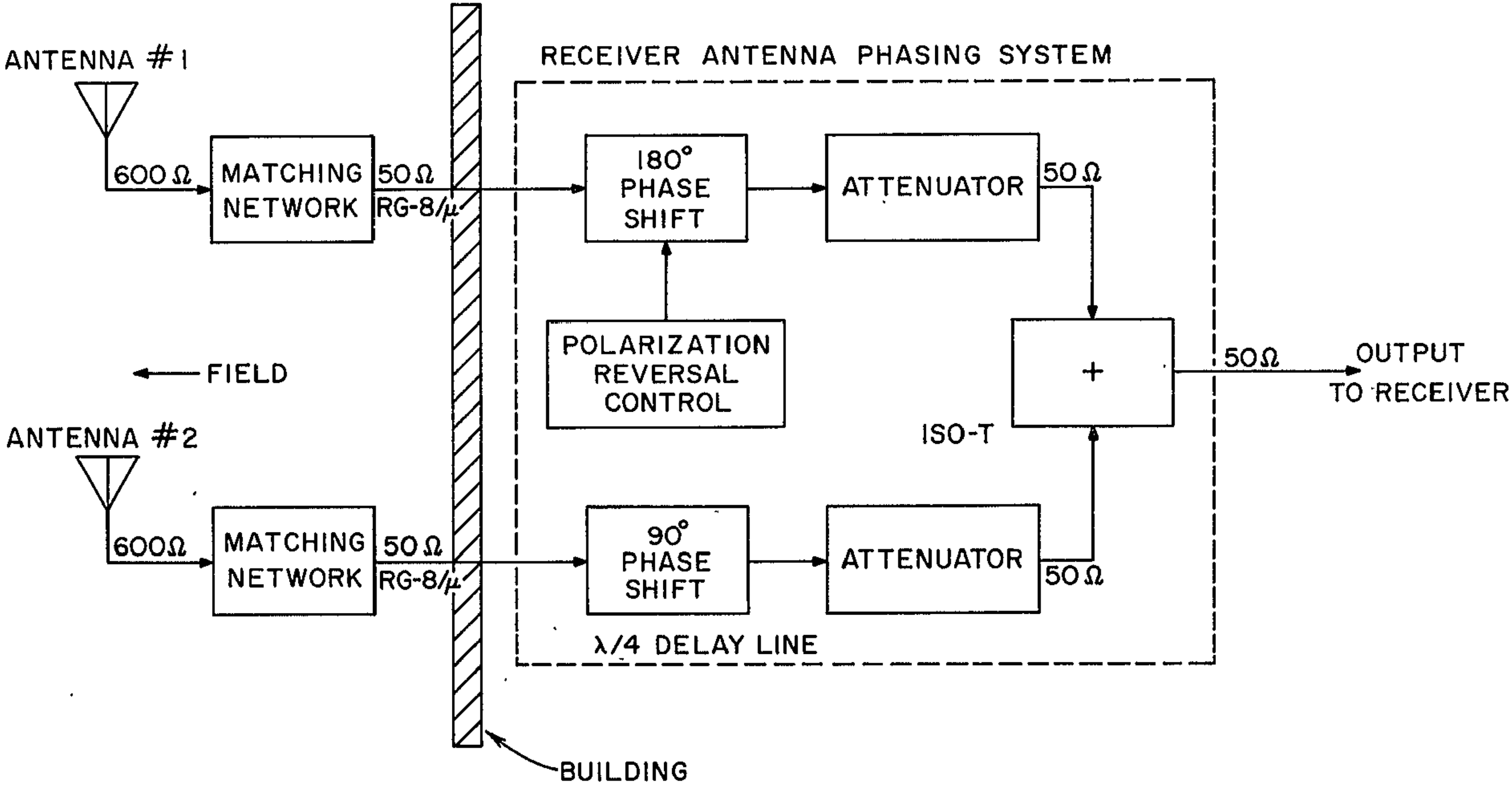
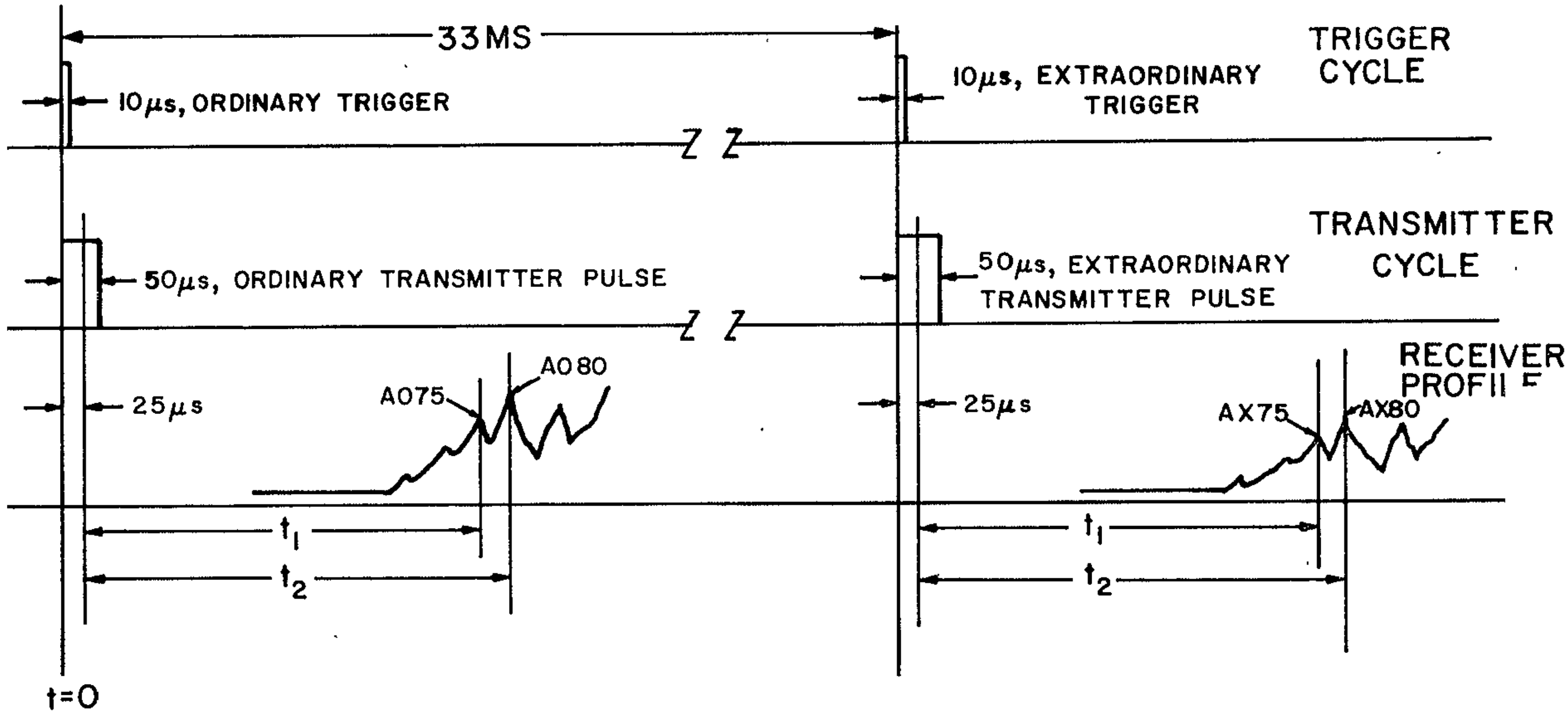


Figure 3.6 Block diagram of the receiver antenna phasing system (after Pirnat, 1968).

4. PROPOSED SOMED DATA COLLECTION SYSTEM

The purpose of the instrumentation designed for SOMED network application is to measure and record parameters necessary for the computation of average electron density between 75 and 80 km. This electron density can be calculated if the collision frequency at 77.5 km, the ratio $|R_x/R_o|$ at both 75 and 80 km, and the ratio A_x/A_o at both 75 and 80 km are all known. A collision frequency profile is assumed; and therefore the collision frequency at 77.5 km is known. The ratio $|R_x/R_o|$ at both 75 and 80 km is also known since this ratio is a function of collision frequency only, as was discussed earlier. The only parameters that remain unknown are two ratios; A_x/A_o at 75 km and A_x/A_o at 80 km. If these can be found average electron density between 75 and 80 km can be dete mined. However, these ratios are uniquely determined if receiver output voltag at times corresponding to 75 and 80 km are known for both the ordinary and extraordinary modes. It is these four voltages which are measured and recorded by the instrumentation which will be described in the following chapters. To understand clearly what these four voltages are, refer to Figure 4.1. The voltage A075 is the receiver output voltage which is caused by the incidence on the receiving antenna of the reflected wave from 75 km for the ordinary transmission. The voltage A080 is the receiver output voltage corresponding to reflection from 80 km for the ordinary transmissions. The voltages AX75 and AX80 are described in similar fashion. However, the receiver output voltage is proportional to receiver input voltage and the proportionality constant can certainly be considered constant over any data recording cycle. Thus, the ratio AX75/A075 is equal to A_x/A_o at 75 km and AX80/A080 is equal to A_x/A_o at 80 km. Thus, the ratio A_x/A_o at both 75 and 80 km are determined.



$$t_1 = \frac{2h_1}{c} = 500\ \mu s \qquad t_2 = \frac{2h_2}{c} = 533\ \mu s$$

Figure 4.1 Measured voltage values per data cycle.

The problem therefore reduces to measuring and recording the four voltage values, A075, A080, AX75, and AX80, for each transmitter cycle. A block diagram of the system which performs this job is shown in Figure 4.2.

The receiver output is applied continuously to the inputs of four sample-and-hold circuits. These four circuits are triggered at the proper times to sample A075, A080, AX75 and AX80 from the receiver output voltage waveform. Since the transmitter cycle time is 2 seconds, each voltage (A075, A080, AX75 and AX80) assumes a new value every two seconds. The result is a stepped waveform with step discontinuities every two seconds. The output of each sample-and-hold circuit is therefore a stepped waveform which represents the variation of that voltage, be it A075, A080, AX75 or AX80, as a function of time. This voltage waveform must then be recorded. To do this the four stepped voltage waveforms (A075(t), A080(t), AX75(t), and AX80(t)) are applied to the drain of four 2N4091 JFET transistors. Every 2 seconds, approximately 20 ms after the last transmitter pulse, the data recording cycle timing control gates first A075(t), then A080(t), then AX75(t), and finally AX80(t) within a period of 0.5 seconds. At the same time that each of the four channels are gated, the data recording cycle timing control sends a punch command to the data recording system and the value of the voltage on that particular channel is punched on paper tape. Thus, every two seconds, four voltage values are punched on the paper tape. The first is the value of A075 sampled from the receiver output, the second A080, the third AX75, and the fourth AX80.

In discussing the sample-and-hold circuits, it was mentioned that these circuits were triggered at such a time as to sample A075, A080, AX75, and AX80 from the receiver waveform. The trigger pulses are derived from two

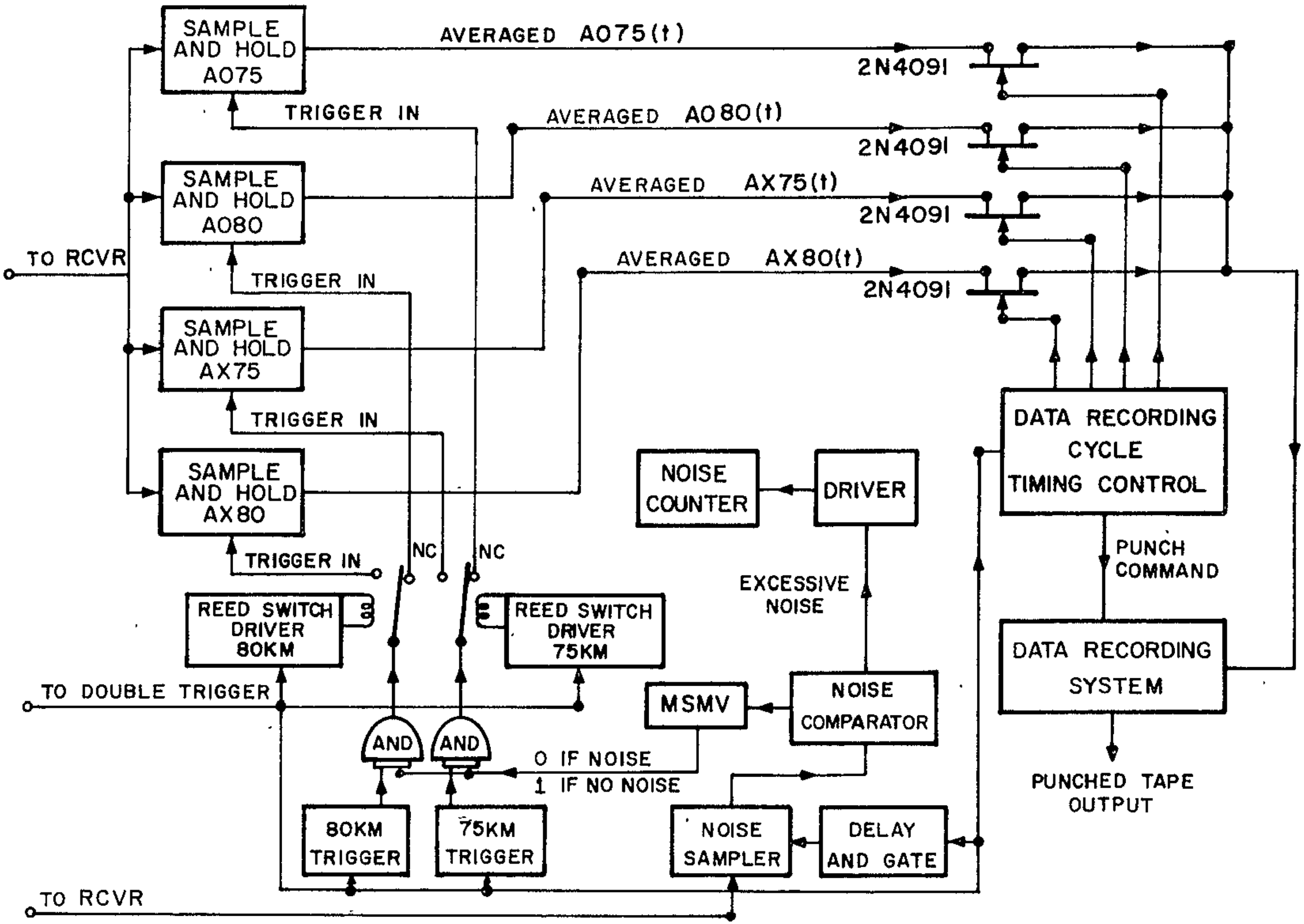


Figure 4.2 Block diagram of SOMED facility.

triggering circuits, one for 75 km and one for 80 km. These two circuits are in turn triggered by the reference double trigger which supplies the two trigger pulses, spaced at 33 ms for the transmitter. Since the same trigger circuit is used for both the ordinary and the extraordinary mode, a switching mechanism is necessary to switch the trigger outputs between the ordinary and the extraordinary sample-and-hold circuits. The switching is done utilizing two reed switches which are driven by two driver circuits; these are also triggered from the reference double trigger.

At times the voltage waveform at the output of the receiver may be contaminated with excessive noise such as that due to lightning flashes in the immediate area of the partial-reflection system, or even hundreds of miles away. To prevent such noise from contaminating the data obtained from the system, a noise discriminator circuit is utilized which checks the receiver voltage amplitude at a time when it should be minimum; that is, at a time when there should be no partial reflections from the mesosphere. This amplitude thus represents noise; and if this amplitude is too large, the outputs of the 75 and 80 km trigger circuits are never allowed to trigger the four sample-and-hold circuits. Hence, these circuits hold the values of A075, A080, AX75, and AX80 that they held during the previous two-second interval. When the noise level returns once again to an acceptable level, triggering proceeds in a normal fashion. Each time noise is present, however, a driver is triggered which actuates a counter. Thus, the number of eliminated data cycles is continuously monitored; and this number is an indication of the quality or reliability of the data taken during a particular recording interval.

5. CIRCUIT DESCRIPTION

5.1 The Sample-and-Hold Circuits

Figure 5.1 shows the schematic diagram of the sample-and-hold circuit. For the system, four of these circuits are required, one each to sample the four voltages A075, A080, AX75, and AX80. The circuit for each is identical; however, each is triggered at a different time. The sample-and-hold circuit for A075 is triggered when the voltage on the receiver output corresponds to A075. Similar statements apply to the sample-and-hold circuits for A080, AX75 and AX80.

Suppose that a signal f(t) is applied to the signal input terminal, A, of one of the sample-and-hold circuits as shown in Figure 5.1. The μA741, an integrated circuit operational amplifier which acts as a buffer and as a unity gain voltage follower, tracks this voltage and applies it to the drain of the 2N4091 JFET transistor. As a result, if the circuit is triggered at t_o, C_1 charges to the voltage applied to the input of the μA741 at t_o, namel $f(t_o)$. Suppose a triggering pulse of the type shown in Figure 5.1 is applie to the trigger input. At t_o the channel of the JFET transistor opens and C_1 is allowed to charge. At $t = t_o + \tau$ the channel is closed and the capacitor C_1 holds the value of voltage to which it was charged at $t = t_o + \tau$. Call t voltage $V_c(t_o+\tau)$. If f(t) varies slowly compared with τ we can say that $f(t_o+\tau) \approx f(t_o)$. And, if the charging time constant of C_1 is small compared τ, then $V_c(t_o+\tau) \approx f(t_o)$. The sample-and-hold circuits in the SOMED system have charging time constants of approximately 2 μs whereas τ is 6 2/3 μs. A the receiver output voltage varies slowly enough to justify saying that $V_c(t_o+\tau) = f(t_o)$.

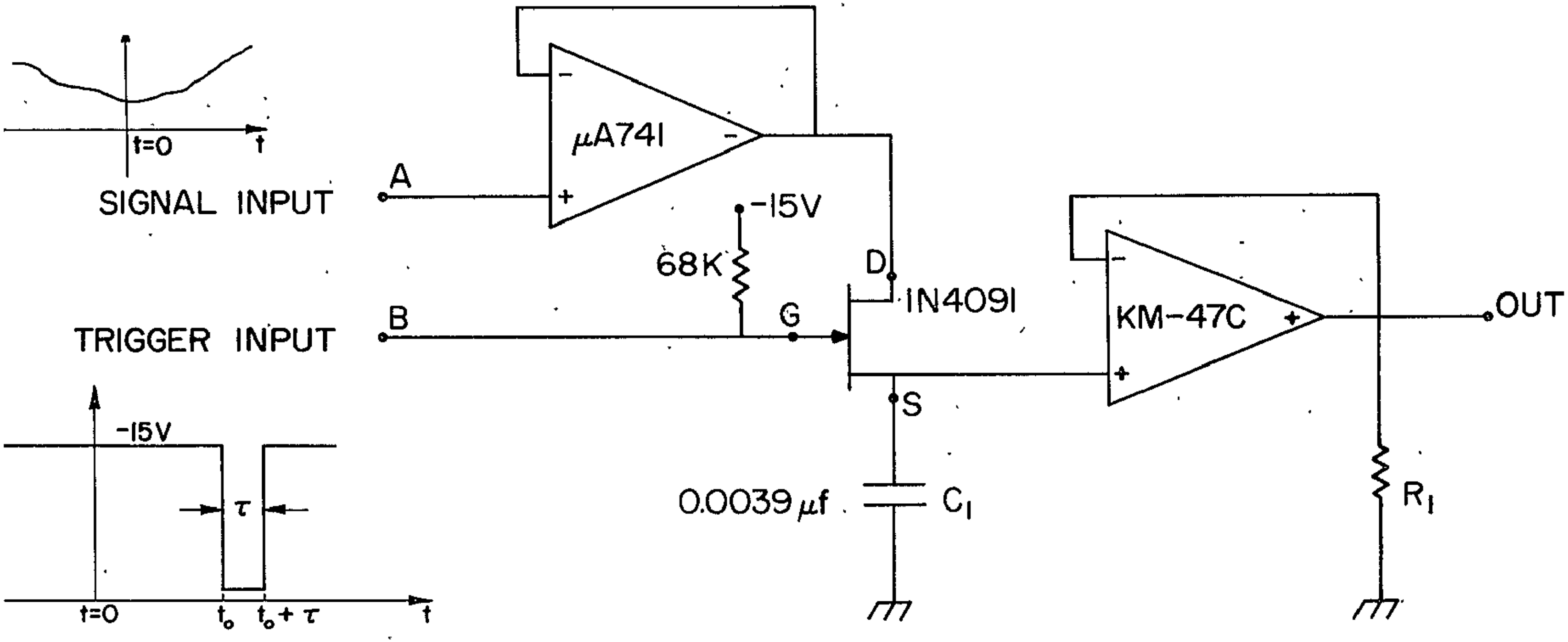


Figure 5.1 The sample-and-hold circuit.

Since each sample-and-hold circuit of the network is gated every two seconds, $V_c(t)$ is a stepped waveform with transitions every 2 seconds. This voltage waveform is applied to a KM-47C unity gain voltage follower. Since the gating pulses applied to C_1 occur at 2 second intervals, the holding time constant for C_1 must be very large compared to 2 seconds. In fact, during the 2-second interval C_1 does not discharge at all. Instead it continues to charge as a result of offset currents in the KM-47C, which is an FET input device with an input impedance of 10^{12} ohms. However, the time required for these offset currents to make a significant change in V_c is greater than 10 seconds and thus creates no problem. Since the voltage on C_1 is a stepped waveform, the output voltage of the KM-47C, which is the output of the sample-and-hold circuit, is also a stepped waveform. With respect to the entire system the outputs of the four sample-and-hold circuits yield A075, A080, AX75, and AX80 as a function of time.

5.2 Noise Discriminator Circuits

Since the reflections from the D region of the transmitted waves cause receiver output voltages which are on the order of magnitude of μvolts and since noise voltage levels are also of this order, a problem of the partial reflection method is the contamination of data by noise. Noise discriminator circuits are used in the SOMED Network instrumentation which prevent contamination by noise such as that caused by lightning; that is, noise which occurs occasionally but which is excessive when it occurs. The presence of this noise is determined as follows. After each transmitted pulse there is a delay before any reflections are received from the D region. During this time the receiver output voltage represents noise. If receiver output voltage is above a certain reference level during this time the noise is considered to be

excessive. This reference level is somewhat arbitrary since disturbances such as lightning flashes take receiver voltages off scale. However, the level must be set at a voltage which is greater than the noise voltages which occur normally during any data cycle.

The noise discriminator circuitry has two inputs. One is the reference double trigger and the other is the receiver output. Through the former input, the reference double trigger triggers delay circuitry each time a RF pulse is transmitted. In response to each trigger pulse, a pulse is thus produced which is delayed from that of the trigger pulse. This delay time corresponds to reception of signals from an altitude of 40 km. Also, the duration of this output pulse is 100 μs. Thus, the end of the pulse corresponds to reception of 55 km signals. This pulse is shaped and then used as a gating pulse to sample receiver output voltage amplitude. This is done by applying the pulse to the gate of a 2N4091 JFET transistor whose drain is connected to the receiver output and whose source is applied to a μA741 voltage follower. When a gating pulse is received, the channel of the 2N4091 transistor opens and the receiver output voltage is then applied to the input of the μA741 voltage follower, which tracks the receiver voltage output and applies it to the input of a μA710 C differential comparator.

If the receiver voltage is greater than the reference voltage of the differential comparator then the comparator makes a state transition. This reference voltage is continuously adjustable from 0 to 5 volts. Thus, if excessive noise is present in the 40 to 55 km region of the receiver output the differential comparator makes a state transition. This transition is then used to trigger two delay circuits. One delay circuit produces a pulse which is applied to two AND gates through which the 75 and 80 km trigger pulses

must pass in order to reach the sample-and-hold circuits of the system; and if triggered, this signal closes the AND gates. Thus, if excessive noise is present in the 40 to 55 km region of the receiver output, the sampling pulses for the 75 and 80 km sample-and-hold circuits are never allowed to reach the sample-and-hold circuits. The sample-and-hold circuits thus retain the values sampled in the previous 2-second period. Since the noise discriminator circuitry is triggered for each transmitted pulse the noise level in the 40 to 55 km region is checked for both the ordinary and the extraordinary mode.

The other delay circuit which is triggered by the state transition of the differential comparator produces an output pulse of 55 ms duration. This pulse is used to trigger a 2N697 transistor which drives an electromechanical counter. Since the pulse used to drive the counter circuitry is of a duration longer than 33 ms (the time spacing between transmitted pulses) the counter can be triggered only once per transmitter cycle. The number present on the counter at the end of each data acquisition period is therefore a record of the number of data cycles which were contaminated by excessive noise. As a result, a qualitative judgement of the quality of the data on any particular data acquisition interval can be made.

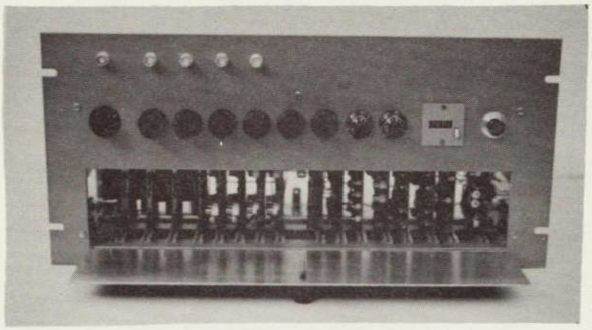
5.3 Data Recording Cycle Timing Control Circuits

As discussed earlier, the output of the four sample-and-hold circuits is a stepped waveform which represents the sampled voltage as a function of time. There are four of these circuits, one each sampling A075, A080, AX75, and AX80. The circuitry described in this section controls the sampling and recording of these four voltage waveforms. The recording is done as follows: approximately 20 ms after the second transmitted pulse of the transmitter cycle, a sample command is given to sample the voltage on

the output of the A075 sample-and-hold circuit. This sampled voltage is applied to the input voltage terminal of the tape punch system. Simultaneously a punch command is also sent to the tape punch system. As a result the voltage value A075 is punched on paper tape in binary coded decimal (BCD) form. Approximately 125 ms later, a voltage value is sampled and recorded from the output of the sample-and-hold circuit for A080. AX75 and AX80 are then recorde both after a delay of 125 ms from the time of sampling of the previous voltage. Thus, every 2 seconds within a period of 0.5 seconds the four voltage values, A075, A080, AX75, and AX80 are recorded. The logic circuitry designed for this task is shown in the general diagrams which follow in the next section and will not be discussed in detail here.

5.4 The Interconnected System

A photograph of the SOMED instrumentation is shown in Figure 5.2. The eight potentiometers shown on the front panel control time delays and pulse output durations which are checked and adjusted frequently. Also included on the front panel is a rotary switch which, through the BNC connector above it, monitors seventeen points scattered throughout the circuitry. In usual operation this provision allows one to check the pulse durations and time delays controlled not only by the potentiometers mounted on the front panel but also by those which are mounted on the printed circuit boards in the interior of the instrument. However, in case of instrument failure the seventeen monitor points provide an excellent means of locating the failing portion of the instrument. Also mounted on the front panel is the electro-mechanical counter mentioned earlier. This counter is set to zero using the mechanical reset button before each data acquisition period. At the end of each period the number on the counter is recorded and represents the number



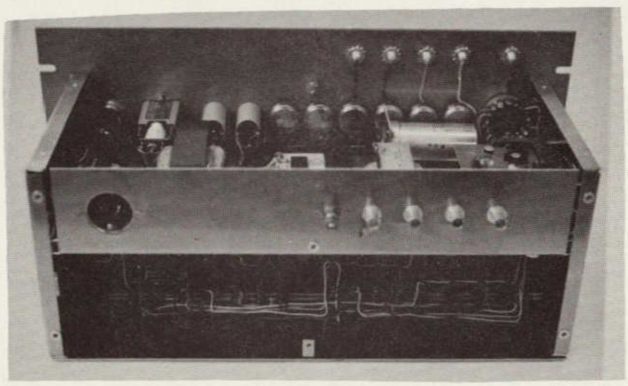


Figure 5.2 SOMED instrumentation.

of data cycles which were contaminated by noise. The hinged panel shown on the front of the instrument makes the sixteen printed circuit boards mounted in the interior of the instrument readily accessible. The power supply circuitry is mounted such that the major portion of the instrument circuitry is shielded from the 60 Hz AC power circuits. This circuitry is also shown in Figure 5.2.

The schematic diagrams for the SOMED instrumentation are shown in Figures 5.3, 5.4, and 5.5. The monitor points of the instrument are numbered from 1 to 17 and correspond to the 17 positions on the rotary monitor switch mounted on the front panel. Figure 5.3 contains the majority of the circuitry associated with the four sample-and-hold circuits of the system. That is, this figure shows the four sample-and-hold circuits; the reed switch driver circuits which control the switching of the 75- and 80-km sampling pulse between the extraordinary and the ordinary sample-and-hold circuits; and the circuitry which produces the 75- and 80-km sampling pulses. Figure 5.4 shows the logic circuitry which controls the data recording of the four voltage waveforms produced by the four sample-and-hold circuits. This circuitry produces the punch commands which are sent to the tape punch unit. This circuitry also produces the gating pulses which "scan" the outputs of the four sample-and-hold circuits in synchronism with the punch commands which are sent to the tape punch unit. The delay circuit in the lower left hand corner of the schematic provides the synchronism between the transmitter and the data recording circuitry to insure that data recording is done only after both transmitter pulses, extraordinary and ordinary, have been transmitted and received. This insures that any noise produced by the tape punch unit does not contaminate the data. The timing of the data recording circuitry

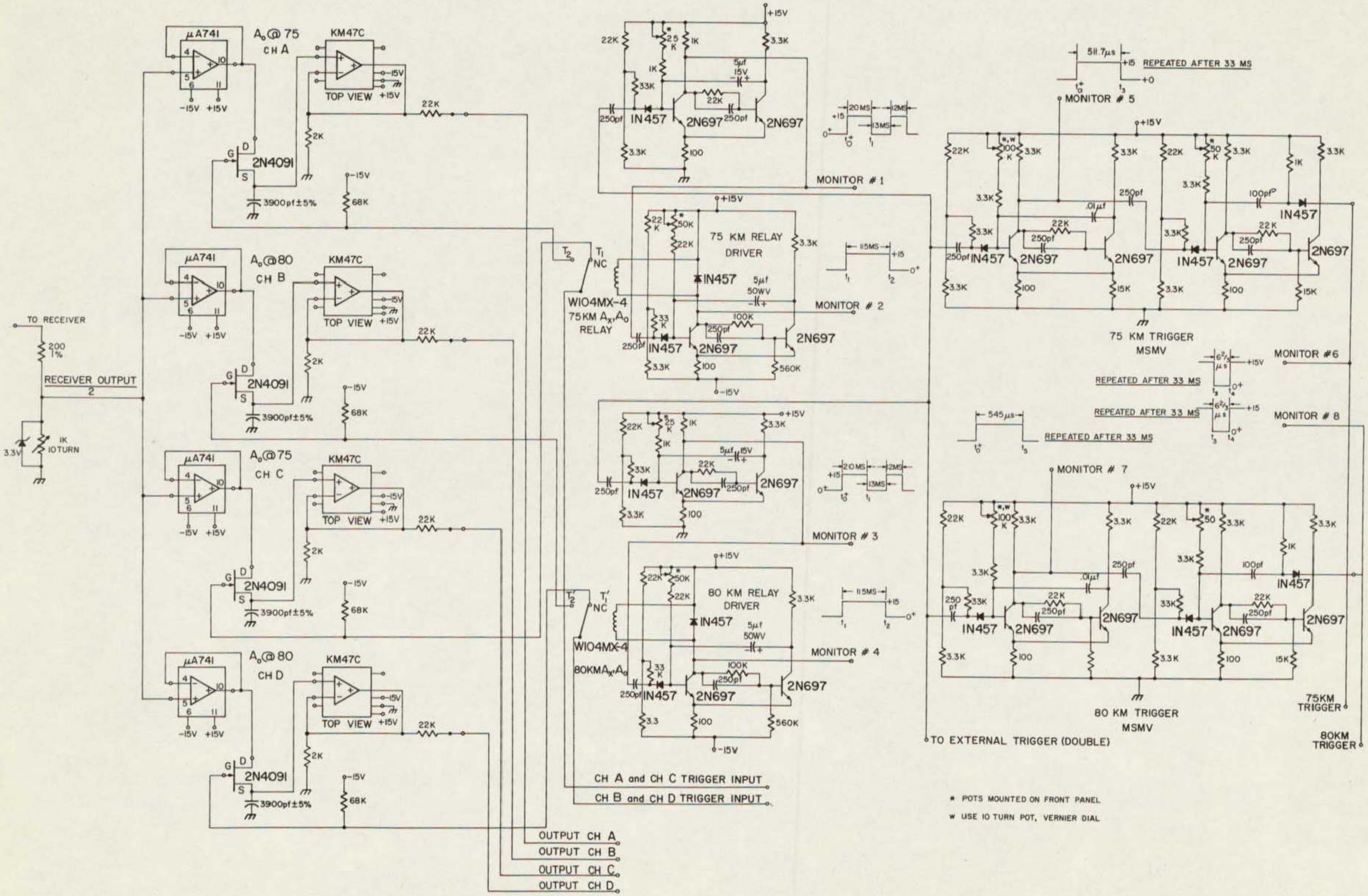


Figure 5.3 Circuitry associated with sample and hold circuits.

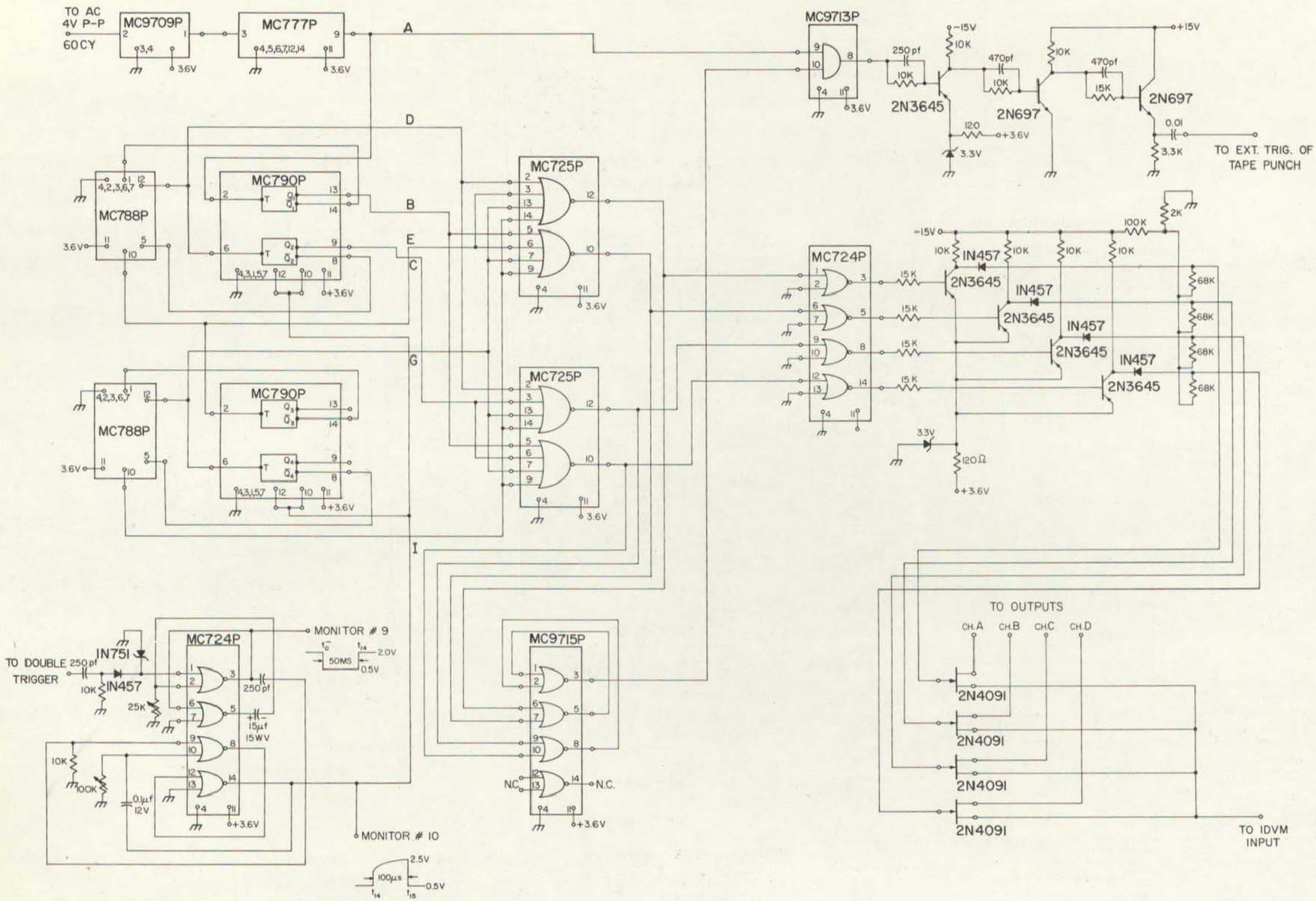


Figure 5.4 Data recording logic circuitry.

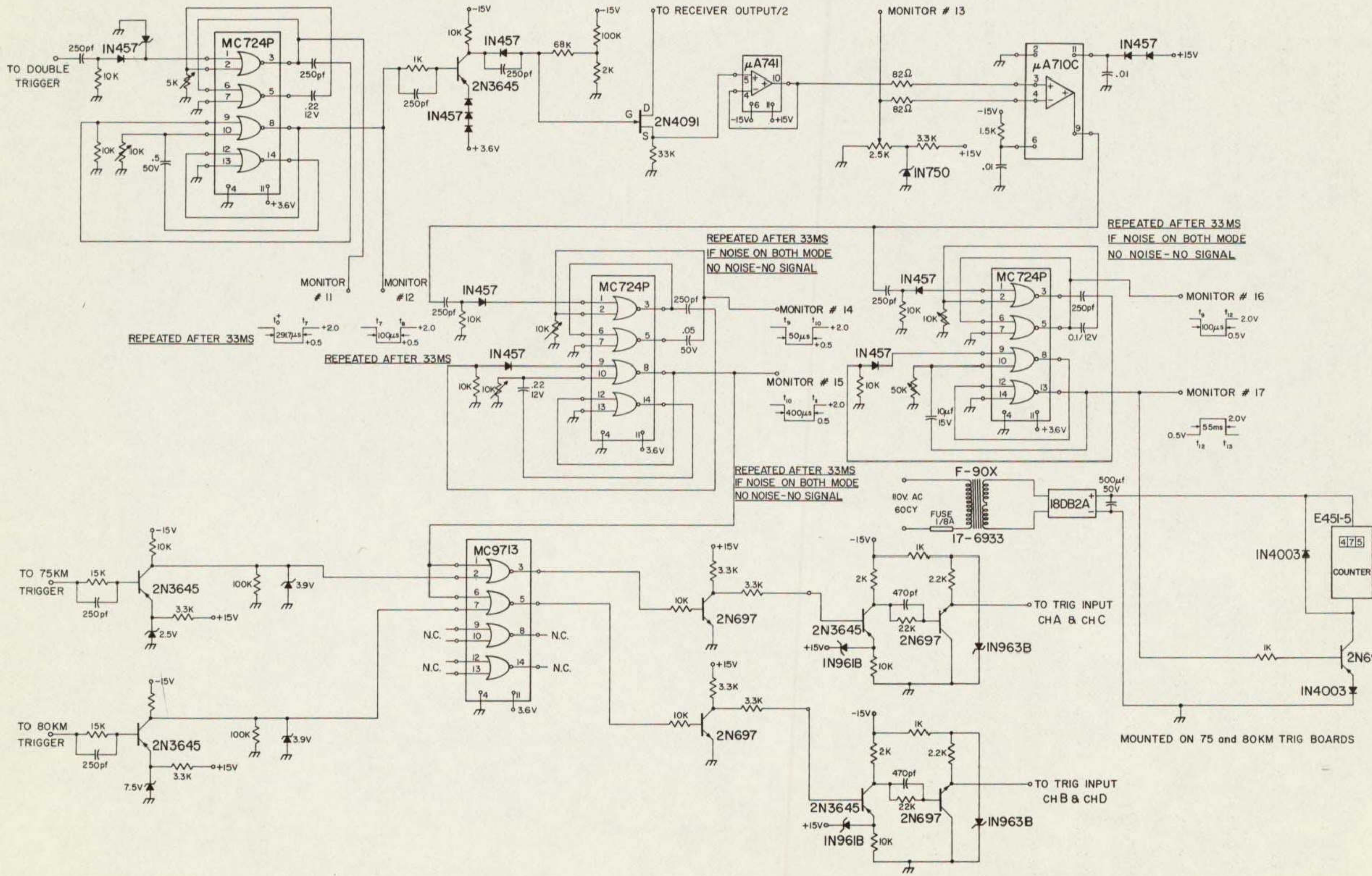


Figure 5.5 Noise discriminator circuitry.

is taken from the 60 Hz power line through the use of a Schmitt trigger circuit and frequency dividers. Figure 5.5 shows the noise discriminator circuitry which has been described in detail in Section 5.2 of this chapter. In the lower center of the diagram is shown the AND gates associated with the noise discriminator circuitry. These gates allow the 75- and 80-km sampling pulses to pass through if noise is not present in the 40- to 55-km region, but block these pulses if noise is present. The circuitry on both sides of the AND gates is not associated with the noise discrimination system but instead are waveform shaping circuitry associated with the 75- and 80-km sampling pulses.

6. DATA RECORDING SYSTEM AND DATA PROCESSING

6.1 The Tape Punch System

The tape punch system used in connection with the SOMED Network instrumentation is the Vidar 1025019 Digital Data Acquisition System. A photograph of the system is shown in Figure 6.1. The system contains three units. These are a Vidar 510 Integrating Digital voltmeter (IDVM), a Tally 420 Tape Perforator, and a Vidar 651-10 System Coupler. The IDVM is a high-sensitivity, floated and guarded, integrating digital voltmeter with the added ability to measure frequency. The instrument offers full scale sensitivities from 10 mv to 1000 VDC and internally generated integration times of 0.01, 0.1 and 1 second. In conjunction with the SOMED Network instrumentation, the IDVM integration time is set at 0.01 seconds. This means that any voltage measured by the IDVM is averaged over 0.01 seconds. Also incorporated in the IDVM is an external trigger unit. This unit provides for externally controlled timing of the Vidar system. This provision is also used by the SOMED instrumentation. The punch command pulses produced by the data acquisition control circuitry of the SOMED instrumentation are applied to the Vidar tape punch system through the external trigger terminal of the external triggering unit. Each time a punch command is received, the voltage on the input to the IDVM is sampled and the tape punch unit of the system punches the voltage sampled by the system. Four punch commands are sent during each transmitter cycle. The data acquisition control circuitry of the SOMED instrumentation produces these punch commands and "scans" the outputs of the four sample and hold circuits in such a way that on the first punch command the voltage waveform A075 is present on the voltage input terminal of the IDVM. On the second

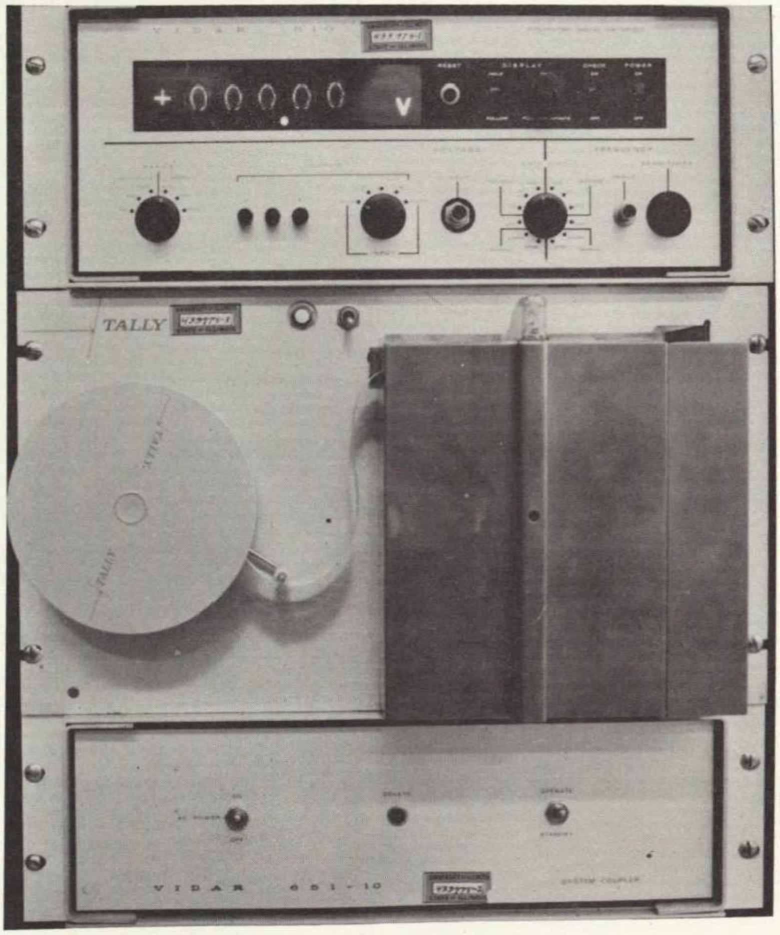


Figure 6.1 Digital data acquisition system.

punch command the voltage waveform A080 is present on the input to the IDVM. Similarly AX75 and AX80 are present on the third and fourth punch commands. Thus, every two seconds four voltage values are punched on paper tape in BCD form. These voltages are A075, A080, AX75, and AX80.

The Vidar 651-10 System Coupler provides format conversion and most timing functions for the Vidar Digital Data Acquisition System. As a result it is a key instrument of the system.

The Tally 420 Tape Perforator is capable of perforating the eight-level tape used in the system at rates up to 60 Hz. Output data for the system are recorded in IBM eight-level code. Polarity, four digits of data, and end-of-line indications are recorded using the code and format shown in Figure 6.2. Odd parity indications are also punched by the system.

6.2 Data Processing

The data taken during any data acquisition period are recorded on paper tape. These data must then be processed so as to yield electron density at 77.5 km. The G-20 computer at the University of Illinois is used for this task. The program used to control data processing is presented in detail in the Appendix and is summarized as follows.

The paper tape reader is set over the first character punched on the paper tape. This character is always a positive sign. The computer then receives a "read" command and the first 24 characters are read off the paper tape. These twenty-four characters contain all the information recorded during the first transmitter cycle. These characters are then decoded and the value of A075, A080, AX75, and AX80 recorded on the first transmitter cycle are computed. Then, the computer receives a second "read" command and 24 more characters are read off the paper tape. These characters contain the data

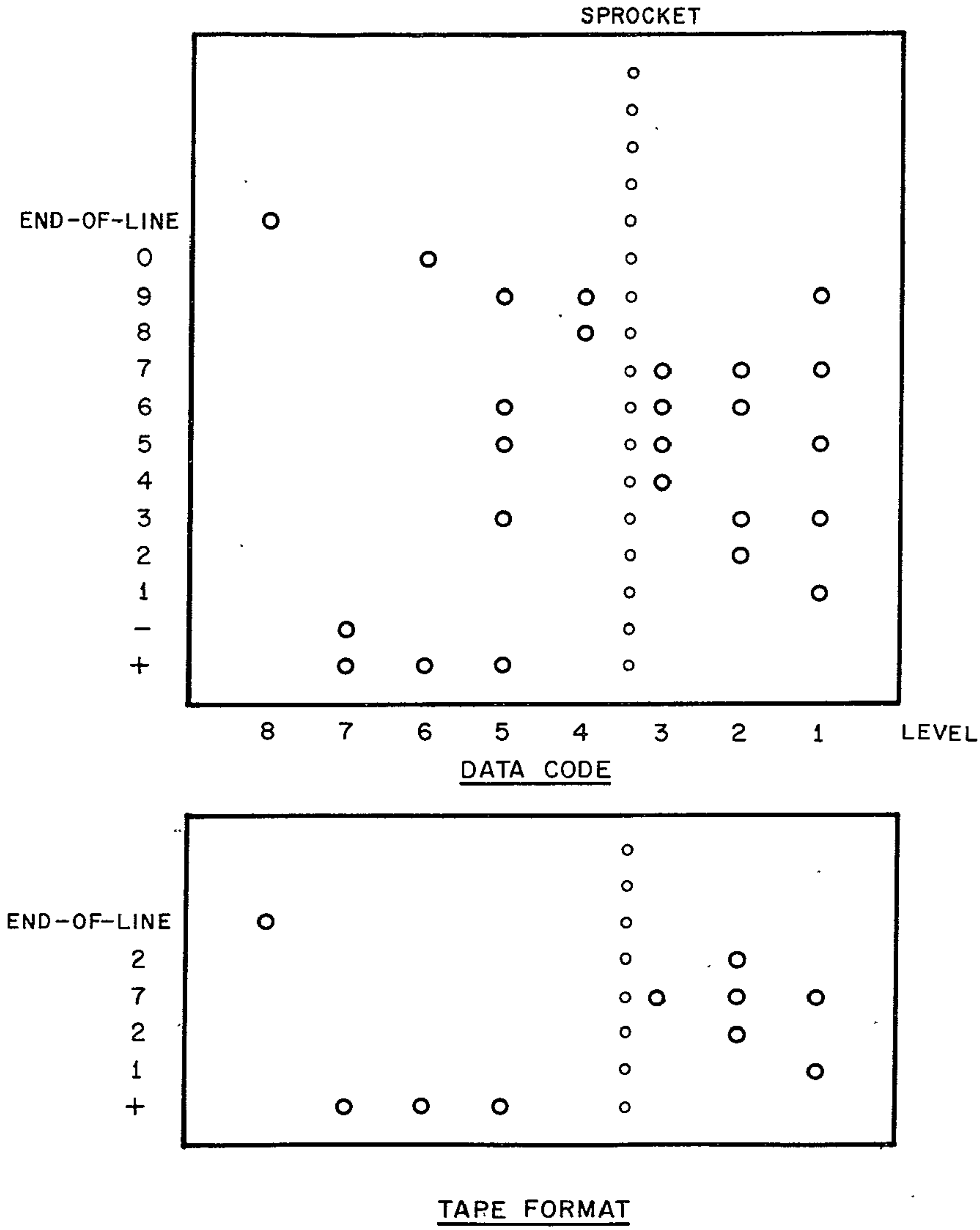


Figure 6.2 Data code and recording format.

recorded during the second transmitter cycle. This process is continued until thirty groups of twenty-four characters are read off the paper tape. The thirty values computed for each of the voltages A075, A080, AX75, and AX80 are then summed and the ratios AX75/A075 and AX80/A080 are formed. From these ratios a value of electron density is calculated. This electron density and the averaged values of AX75/A075 and AX80/A080 are then printed. Since each group of twenty-four characters represents two seconds of real time and since data contained in thirty groups of twenty-four characters are used to compute the amplitude ratios and the electron density, the values of these ratios and the electron density printed represent a time average of one minute. This cycle is repeated until all the data punched on the paper tape has been processed. Thus, for every minute of data recorded a value for AX75/A075, AX80/A080, and electron density at 77.5 km is computed which represents an average of thirty data points. Initial data acquisition intervals will be limited to thirty minutes. Therefore, the output of each experiment will be a profile of AX75/A075 versus time, a profile of AX80/A080 versus time, and a profile of electron density at 77.5 km versus time. Each time variable will run from 0 minutes to 30 minutes, with a data point for each minute interval.

The program has also been written so as to compute a daily average of AX75/A075, AX80/A080, and electron density at 77.5 km. For a thirty minute data acquisition interval this average would therefore represent an average over 900 data points.

It was mentioned earlier that an end-of-line character was punched as the last character of each "word" punched on the paper tape. The Vidar instrumentation uses this end-of-line character to reset itself for the following punch command. Occasionally a zero is punched for the end-of-line character. When this

happens the instrumentation does not reset itself and an extra group of characters are read in. To compensate for this, the computer program checks end-of-line characters for this occurrence. This check is made for every group of twenty-four characters read in by the computer. A flow chart of the checking process is shown in Figure 6.3. The KK(I) represent the twenty-four characters read in by the computer, where I=1,2,3,....,24. Since end-of-line characters occur as the 6th, 12th, 18th, and 24th characters, KK(6) = kk(12) = KK(18) = KK(24) = 128 if all end-of-line characters are punched properly. If one of these KK's is not 128 it is checked to determine whether or not it is 32 (a zero). If it is 32 this means that an extra group of six characters was punched by the tape punch system. The computer then reads in and discards six characters to compensate for the error. If two errors of this type occur within a twenty-four character group the run is aborted.

Occasionally, a punch is simply skipped. The end-of-line characters are also checked for this occurrence. If the character punched in the end-of-line position is not an end-of-line character or a zero, the character punch position just before the end-of-line position is checked for occupation by an end-of-line character. If it is occupied by an end-of-line character a punch has been skipped. If all end-of-line characters after the displaced end-of-line character are also displaced one position then only one skip has occurred. In this case only twenty-three characters are read in on the next "read" command, and the computer processing continues. If more than one character has been skipped the run is aborted.

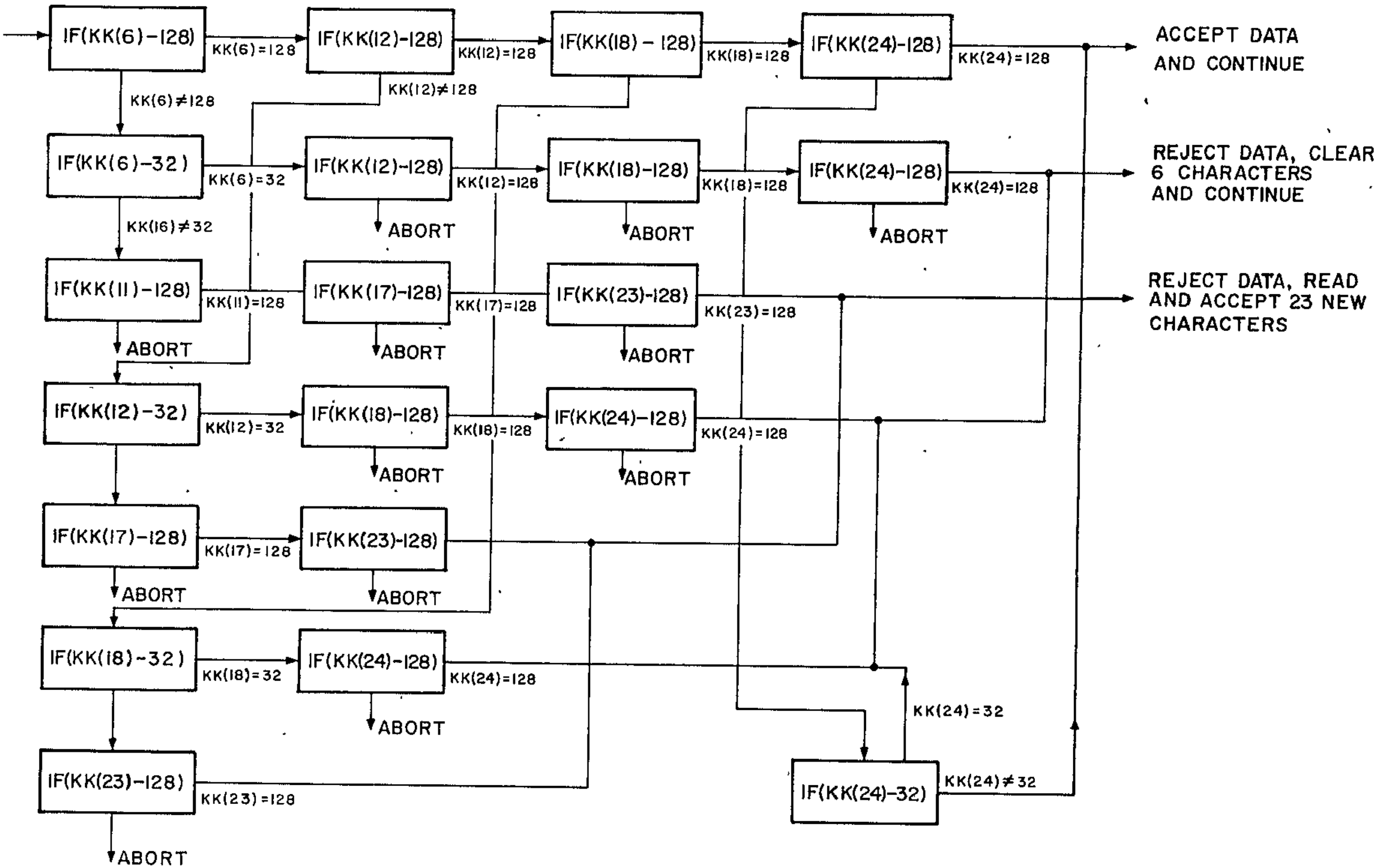


Figure 6.3 Flow chart for character check.

7. OPERATION OF SOMED DATA COLLECTION SYSTEM

Before recording data on any particular day, the instrumentation associated with the SOMED facility should be allowed a thirty minute stabilizing period. This is especially true for the IDVM. Also, the tape punch unit should be operated prior to any data acquisition period, since this insures that the punching mechanism is well lubricated before the actual data acquisition period begins. Also, the SOMED equipment should be turned on and off only in the 1.5 second period between the end of the punching interval and the beginning of a new transmitter cycle. This must be done since the computer is programmed to accept the first voltage value punched on the paper tape as A075. As soon as the tape punch unit is actuated and the first set of four punch commands are received, the voltages, A075, A080, AX75, and AX80, for the first transmitter cycle are punched on paper tape. As the system continues to punch data for the succeeding cycles the punched paper tape is wound on a reel on the tape punch unit. As the first data set is wound on this reel, the first character of this set should be checked so as to insure that it is a positive sign. This is necessary since the computer uses the first character of the first data set as a reference. If it is not present the data on the paper tape will not be interpreted correctly by the computer. After each data acquisition interval the number of frames of data contaminated by noise should be recorded on the paper tape along with the date of acquisition and the time period over which the data were recorded. The end of the tape at which the data begin should be plainly marked "beginning" so that computer personnel can place the data reels on the computer paper tape reader in the correct fashion. Instructions should be included with each tape turned in to computer personnel stating that

the tape should be run at 250 characters/second. The reliability of the paper tape reader is improved when operated at this speed.

As mentioned in Chapter 5, the SOMED instrumentation is equipped with a seventeen position rotary switch which allows one to monitor, through a front panel BNC connector, seventeen test points throughout the SOMED circuitry. The waveforms which should be seen on these monitor points will be discussed in the pages to follow. The seventeen waveforms are shown in Figure 7.1. All waveforms are referenced with respect to the reference double trigger which controls the timing of the entire partial reflection system. This reference is also shown in Figure 7.1. Waveform #1 is associated with the 75 km relay driver circuit, and specifies the delay between a pulse of the reference double trigger and the transition of the 75 km relay from ordinary to extraordinary. This delay is specified as 20 ms; however, any delay less than 33 ms is permissible. Waveform #2 specifies the time duration of the 75 km relay in the extraordinary position. This time delay is defined as 115 ms. However, any delay between 40 and 150 ms would be within reason. Waveforms #3 and #4 are identical to waveforms #1 and #2, respectively, and are used to perform the same functions except that they are associated with the 80 km relay driver. Waveform #5 is associated with the 75 km trigger monostable multivibrator and control the delay of the 75 km sampling trigger pulse from the reference double trigger. The time shown is based on no pulse shift of the transmitted pulse from its triggering pulse (also the reference trigger). Hence, the phase shift of the transmitted pulse from its trigger pulse should be added to this figure to be sure that the 75 km trigger pulse samples at exactly 75 km. Waveform #6 shows the 75 km trigger pulse produced by the 75 km trigger monostable multivibrator. Waveforms #7 and #8 perform similar functions as waveform #5 and #6, respectively.

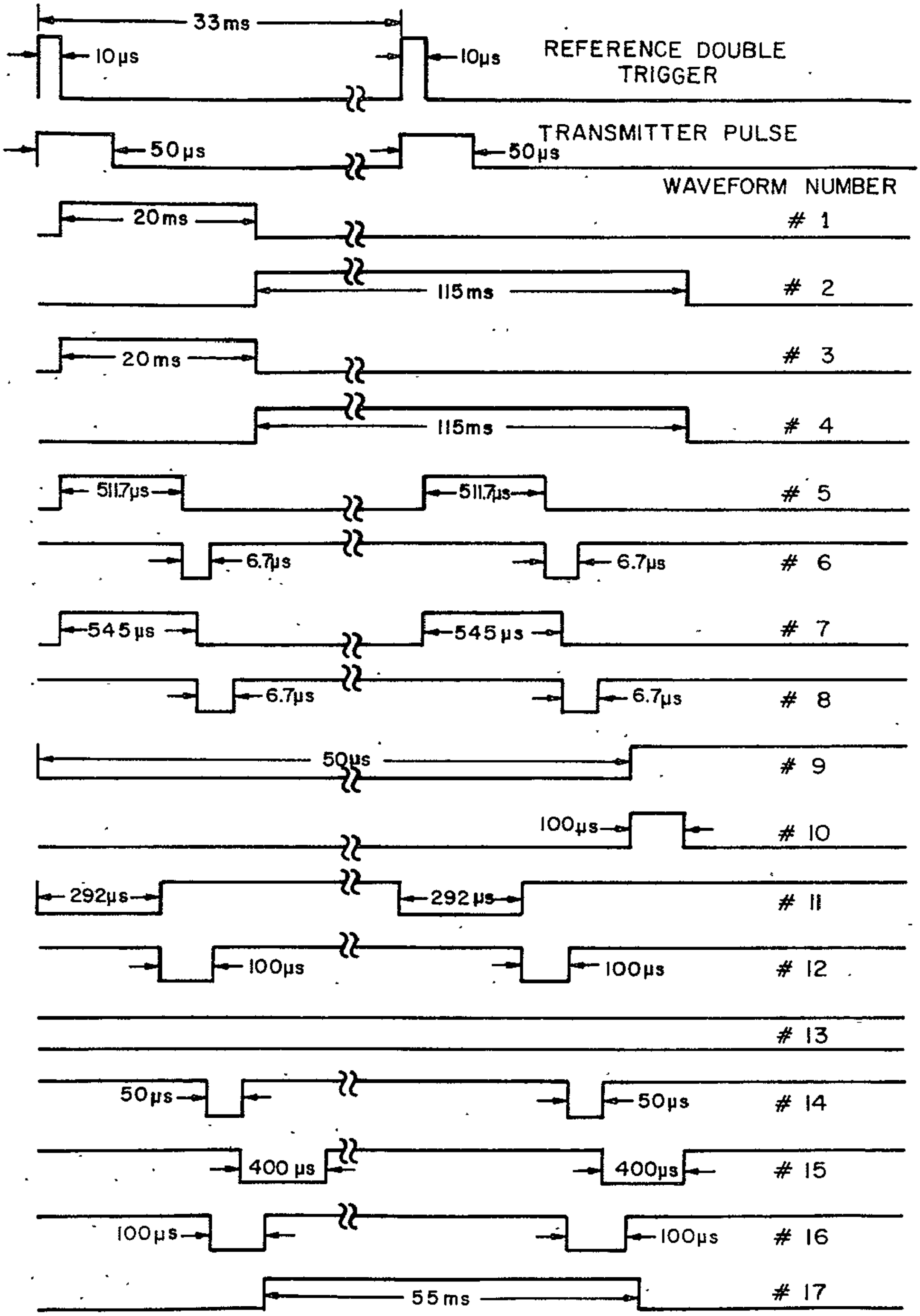


Figure 7.1 Monitor waveforms.

However, the delay is changed so that the sampling pulse corresponds in time to 80 km. The same remarks concerning phase shift of the transmitter apply to waveform #7. Waveform #9 is associated with the data recording logic circuitry. This waveform represents the delay between the first pulse of the reference double trigger and the beginning of the data recording sequence. This delay must be at least 40 ms in order to insure that data recording is accomplished only after data sampling is completed. Waveform #10 shows the pulse which resets the data recording circuitry and initiates the data recording cycle. Waveform #11 is associated with the noise discriminator circuitry and shows the delay between a pulse of reference double trigger and the beginning of the noise sample gate. Waveform #12 shows the duration of this gate. It is the pulse shown in waveform #12 which is used as a gating pulse to sample receiver output voltage in the 40 to 55 km region for noise. Waveform #13 is a constant DC voltage which represents the reference voltage of the noise discriminator circuitry. Waveform #14, which is used as a delay, is only present if the noise discriminator circuitry is triggered by noise in the 40 to 55 km region. In this case the pulse shown in waveform #15, which is delayed by the duration of the pulse shown in waveform #14, is used to block the 75 and 80 km sampling pulses from the sample and hold circuitry. Thus, the time duration of the pulse shown in waveform #15 must begin at a time (controlled by waveform #14) prior to the 75 km sampling pulse and end at a time after to 80 km sampling pulse. Waveforms #16 and #17 provide a delay and a triggering pulse, respectively, for the electromechanical counter. The pulse duration associated with waveform #17 must be at least 20 ms since counter triggering requires a pulse duration of approximately 18 ms.

8. PRELIMINARY DATA

8.1 A-Scope Partial-Reflection Records

Shown in Figures 8.1, 8.2, and 8.3 are typical A-scope photographs of ordinary and extraordinary partial reflections. Here, both the ordinary and the extraordinary reflections are shown superimposed on one another for comparison. The trace which is, in general, of lower magnitude is that of the extraordinary mode. The vertical graticule line to the far left corresponds to the center of the pulse. Also, since the horizontal line sweep is set at 100 μs per division, each cm corresponds to 15 km. Thus, the center vertical graticule line corresponds to 75 km. This means that the amplitude of reflections which are produced at 75 km are proportional to the magnitude of the voltage at the center vertical graticule. It is this voltage which is sampled to obtain reflected amplitude at 75 km. All the A-scope photographs shown were taken between 1400 and 1420 CST on April 28, 1970.

Partial reflections were sometimes observed at altitudes as low as 60 km although, in general, reflections seemed to be restricted to altitudes greater than 70 km. Also, most data cycles showed a peak in the 77 to 78 km region, as revealed in the A-scope records shown in Figures 8.1, 8.2, and 8.3. The height corresponding to such a peak is often called a "preferred height".

8.2 Accuracy Limits of Averaging Procedure

Values of A075, A080, AX75, and AX80 are recorded every two seconds by the SOMED instrumentation. Thus, in one minute 30 values of each voltage are obtained. An average of each voltage is then obtained and assumed to represent the mean of each voltage for that one-minute interval. The accuracy of this assumption is examined here.

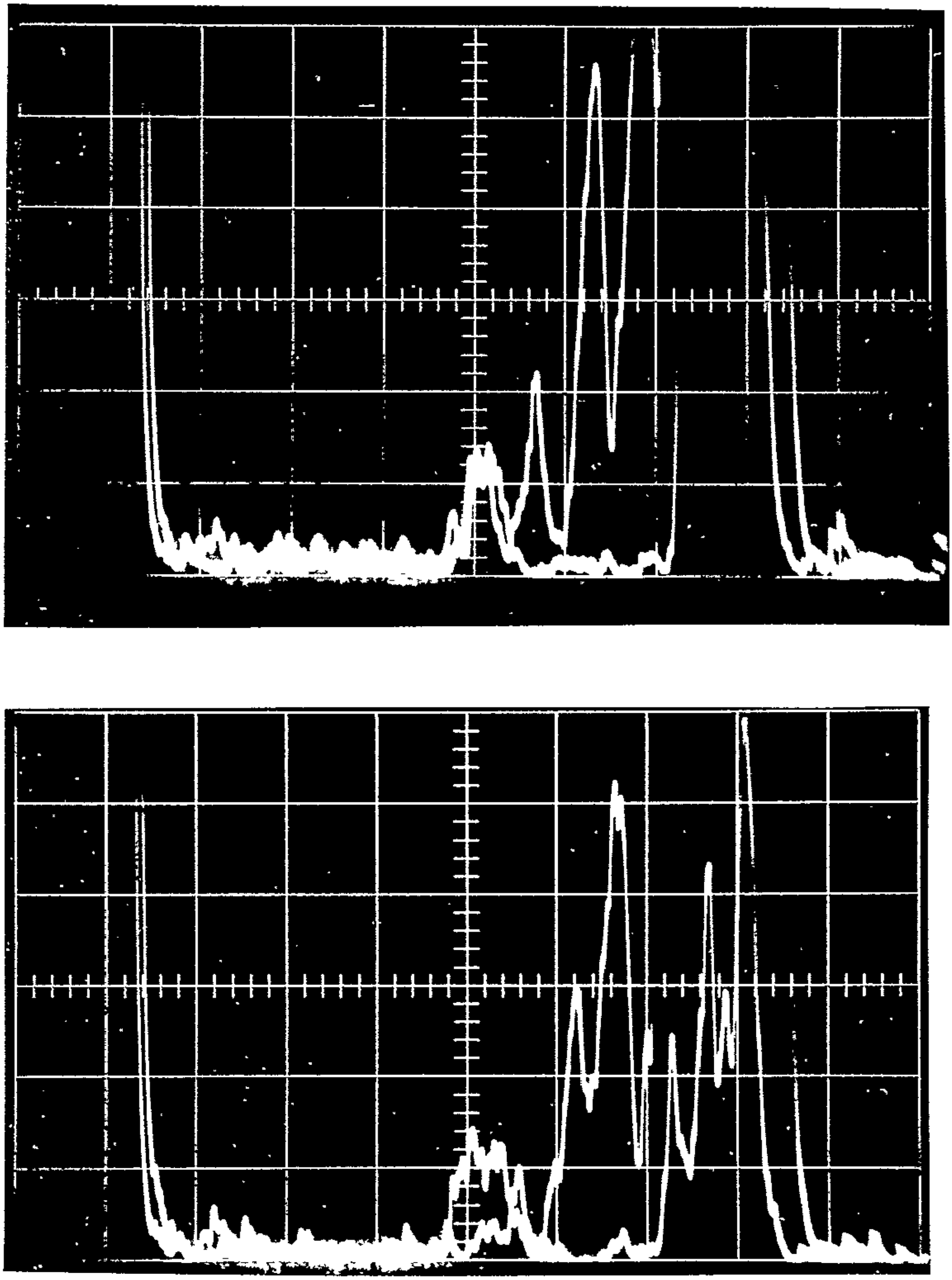


Figure 8.1 Data records.

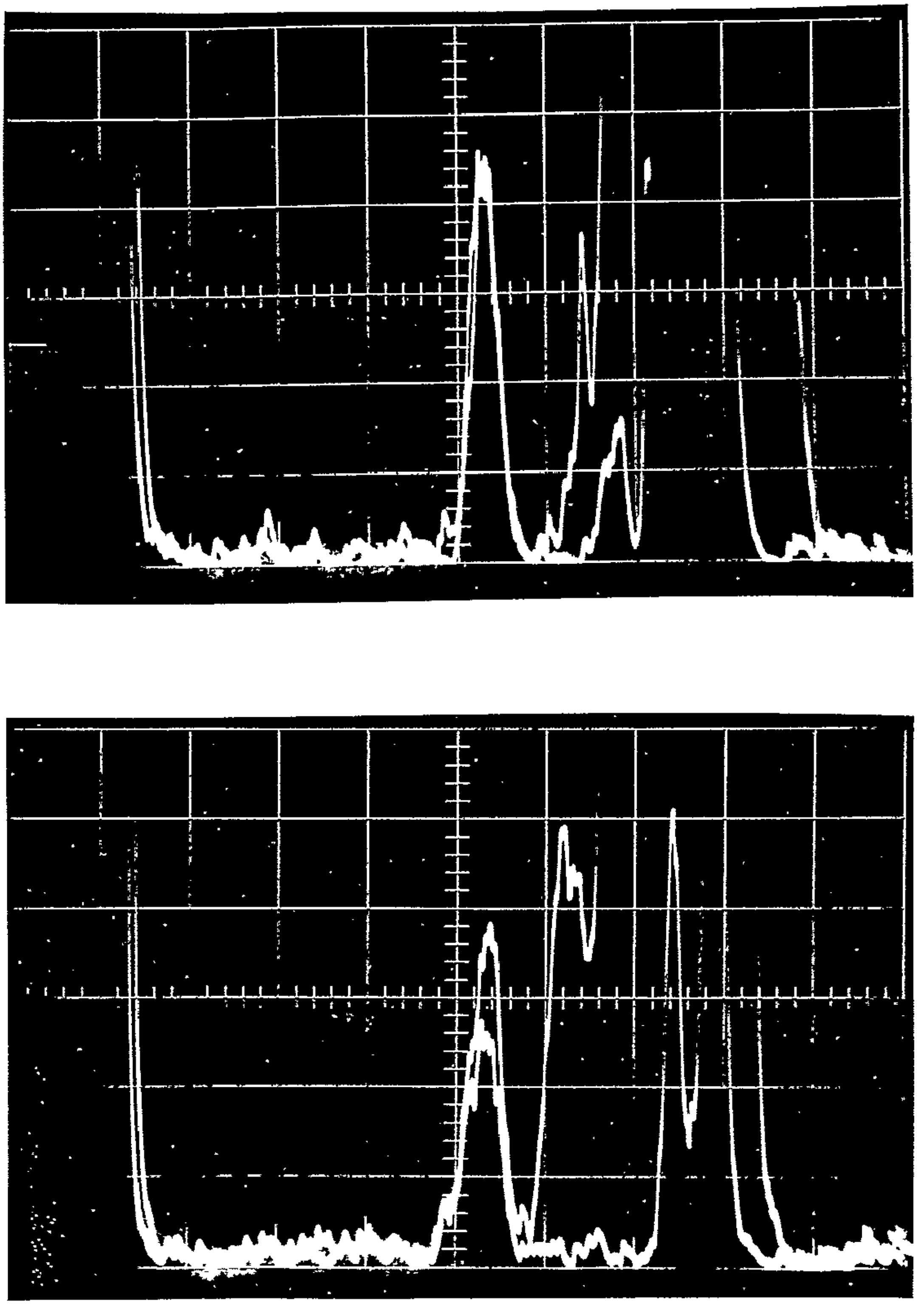
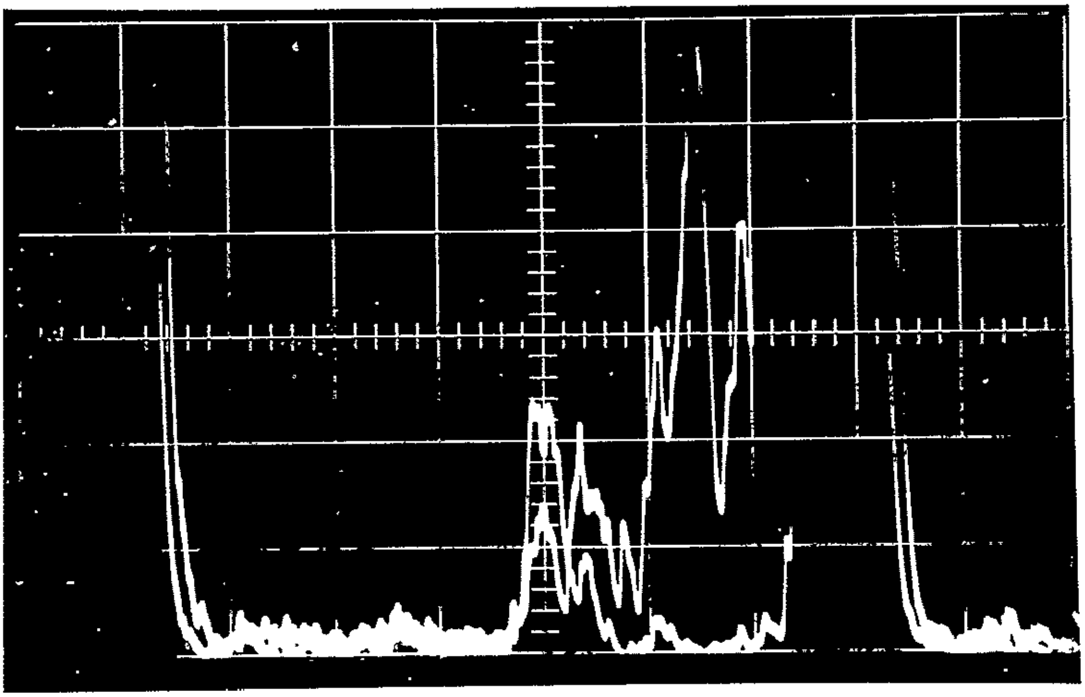


Figure 8.2 Data records.



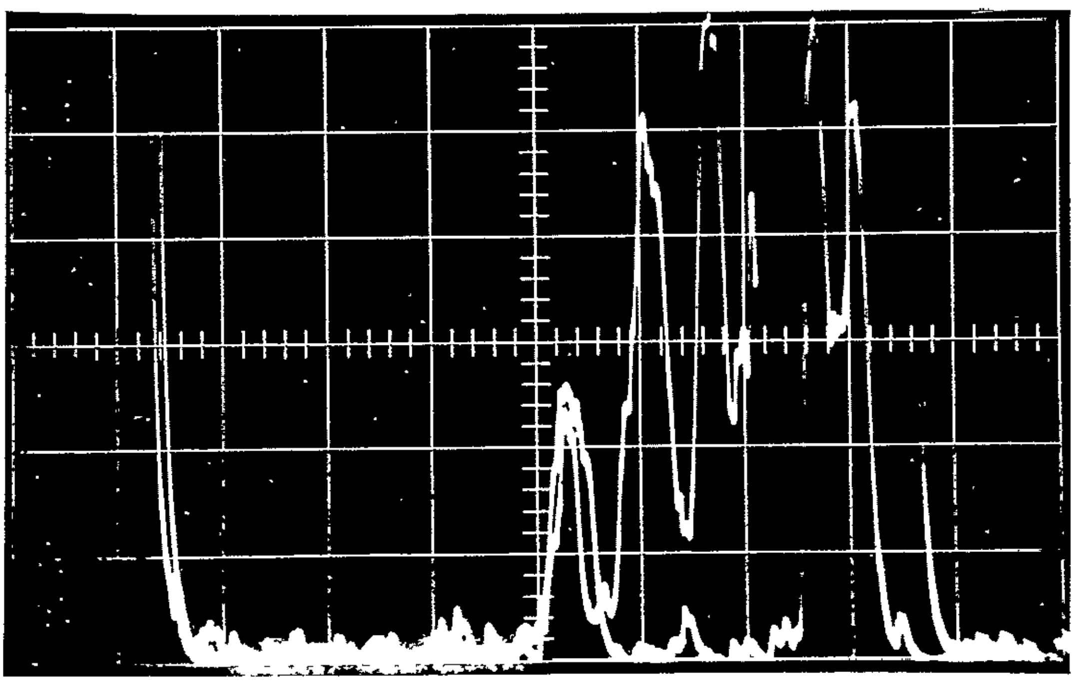


Figure 8.3 Data records.

Consider only one voltage, say A075, and let it be denoted by X. Also, let its value during the first data cycle be given by X_1. Similarly, its value during the second data cycle should be denoted by X_2, and so on. Let y represent the average of the X_i's over N data cycles.

$$y = \frac{X_1 + X_2 + X_3 + \ldots X_N}{N}$$

The voltages with which we are concerned vary between 0 and 3.0 volts with an unknown probability density distribution. Therefore, assume that X_i is constrained between 0 and 3 with probability one and that it is uniformly distributed in that interval for all i. Also, assume that the X_i's are independent. The central limit theorem (Papoulis, 1965) states that as N increases, the probability density of y, $f_y(y)$, will tend to a normal curve regardless of the shape of the probability density of the X_i, $f_X(X)$. Therefore, to determine the probability density, $f_y(y)$, we need only find its variance and its mean. The mean of a sum of N identically distributed random variables is N times the mean of one. Therefore,

$$M_y = \frac{1}{N}[N\ M_X] = M_X$$

where

M_y = the mean of the random variable y,

M_X = the mean of the random variable X.

Thus, the mean of y equals the mean of X. Also, the variance of a sum of independent identically distributed random variables equals the sum of the variance. Therefore,

$$\sigma_y^2 = \frac{1}{N^2}\,[N\sigma_X^2] = \frac{\sigma_X^2}{N}$$

where

σ_y^2 = the variance of y,

σ_X^2 = the variance of X.

The probability density of y can now be written as

$$f_y(y) = \left[1/\sqrt{2\pi\sigma_X^2/N}\right] \exp\left[-(y-M_X)N/2\sigma_X^2\right]$$

In the data presented in this chapter, N is assigned the value 30. Also, since the X_i are assumed to be uniformly distributed in the interval 0 to 3.0, $M_X = 1.5$ and $\sigma_X^2 = 0.75$.

Tchebycheff's inequality can now be used to find a bound on the difference between y and the mean of X, M_X. The inequality is given as follows:

$$P\{|y - M_y| \geq k\sigma_y\} \leq \frac{1}{k^2}$$

But $M_y = M_X$, therefore

$$P\{|y - M_X| \geq k\sigma_y\} \leq \frac{1}{k^2}$$

For N = 30

$$\sigma_y^2 = \frac{\sigma_X^2}{30} = \frac{0.75}{30} = 0.025$$

This implies,

$$P\{|y - M_X| \geq k(0.158)\} \leq \frac{1}{k}$$

Now let k = 3.0; this implies

$$P\{|y - M_X| \geq 0.47\} \leq \frac{1}{9} = 0.11 .$$

This shows that y is within 0.47 of the mean of X with a probability of greater than or equal to 0.89. Thus, under the assumptions stated, the averages of A075, A080, AX75, and AX80 are within 0.47 volts of their means with a probability greater than or equal to 0.89.

8.3 Presentation of Results

Since completion of work on the SOMED facility at Urbana, Illinois, data have been recorded routinely during most working days. Presented in this section are the results obtained on six days during March and April of 1970. These are shown in Figures 8.4 to 8.15. The collision frequency model used in the calculations is an updated version of the collision frequency model used by Pirnat (1968). Collision frequency is assumed to be proportional to pressure, the proportionality constant being $K = 7.0 \times 10^5$ where pressure is given in Newtons per square meter. Pressure values are taken from a Reference Atmosphere (CIRA, 1965). For the days April 21, 22, 23, and 25, graphs of the amplitude ratios as a function of time are plotted. For the days April 28 and May 1, plots of the four voltage waveforms, A075, A080, AX75, and AX80 are shown as well. All graphs show a thirty-minute time span except that of May 1, 1970, which covers a span of 15 minutes. On each graph of electron density and on each graph of amplitude ratio a mean is given which represents an average over all data points obtained in the 30-minute data acquisition period (15 minutes for the data of May 1, 1970), and thus represents an average over 900 samples (450 in the latter case).

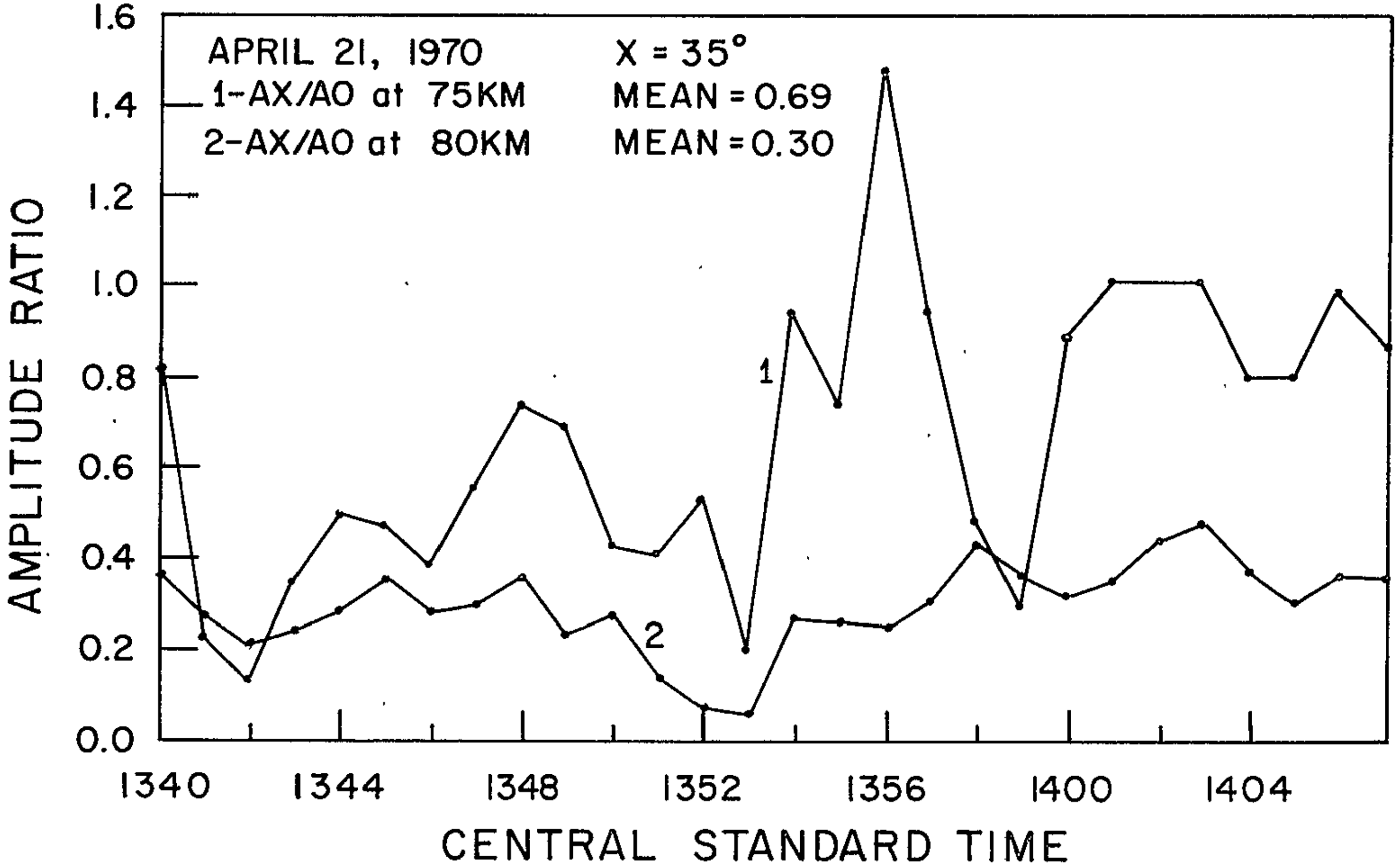


Figure 8.4 Amplitude ratios, April 21, 1970.

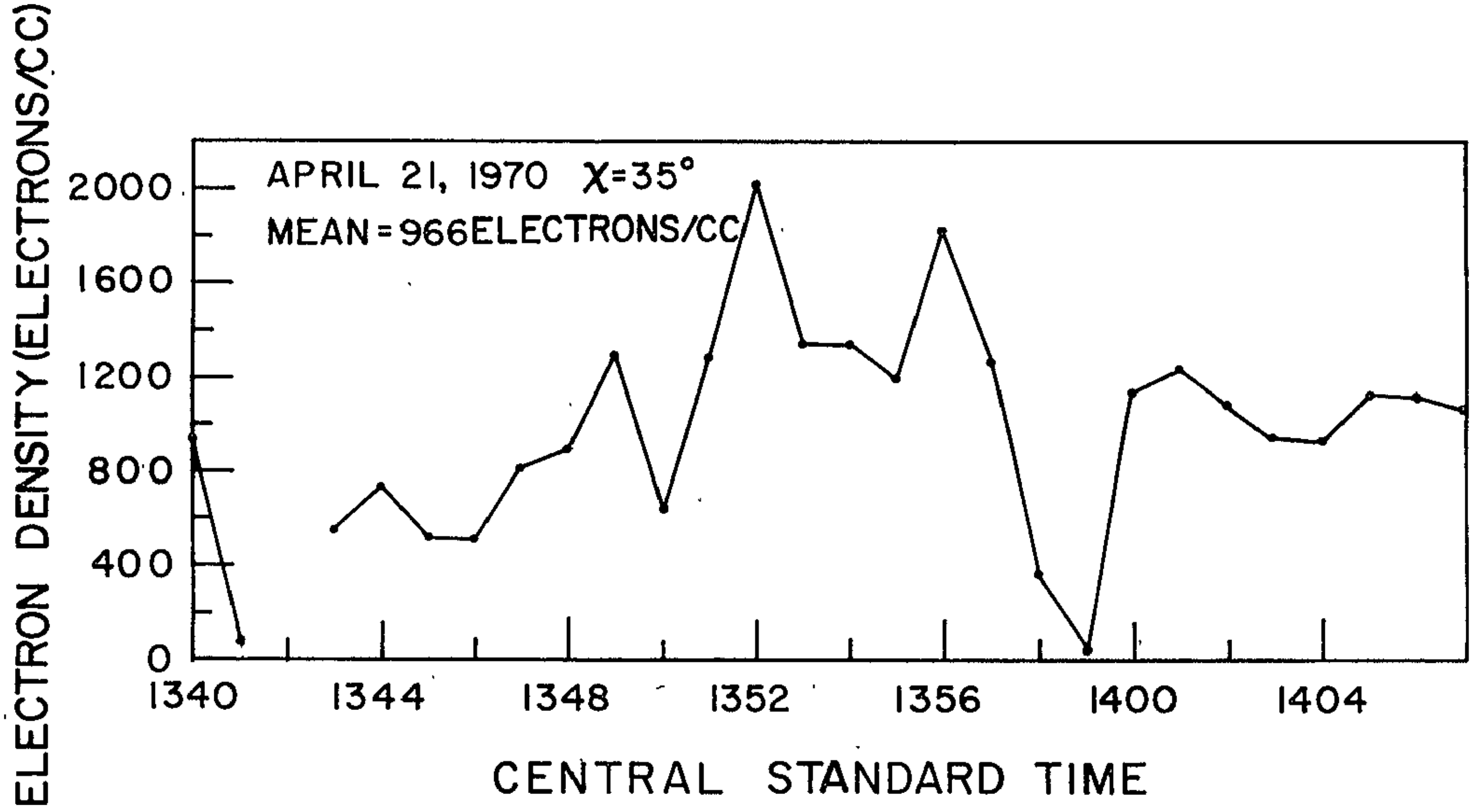


Figure 8.5 Electron density versus time, April 21, 1970.

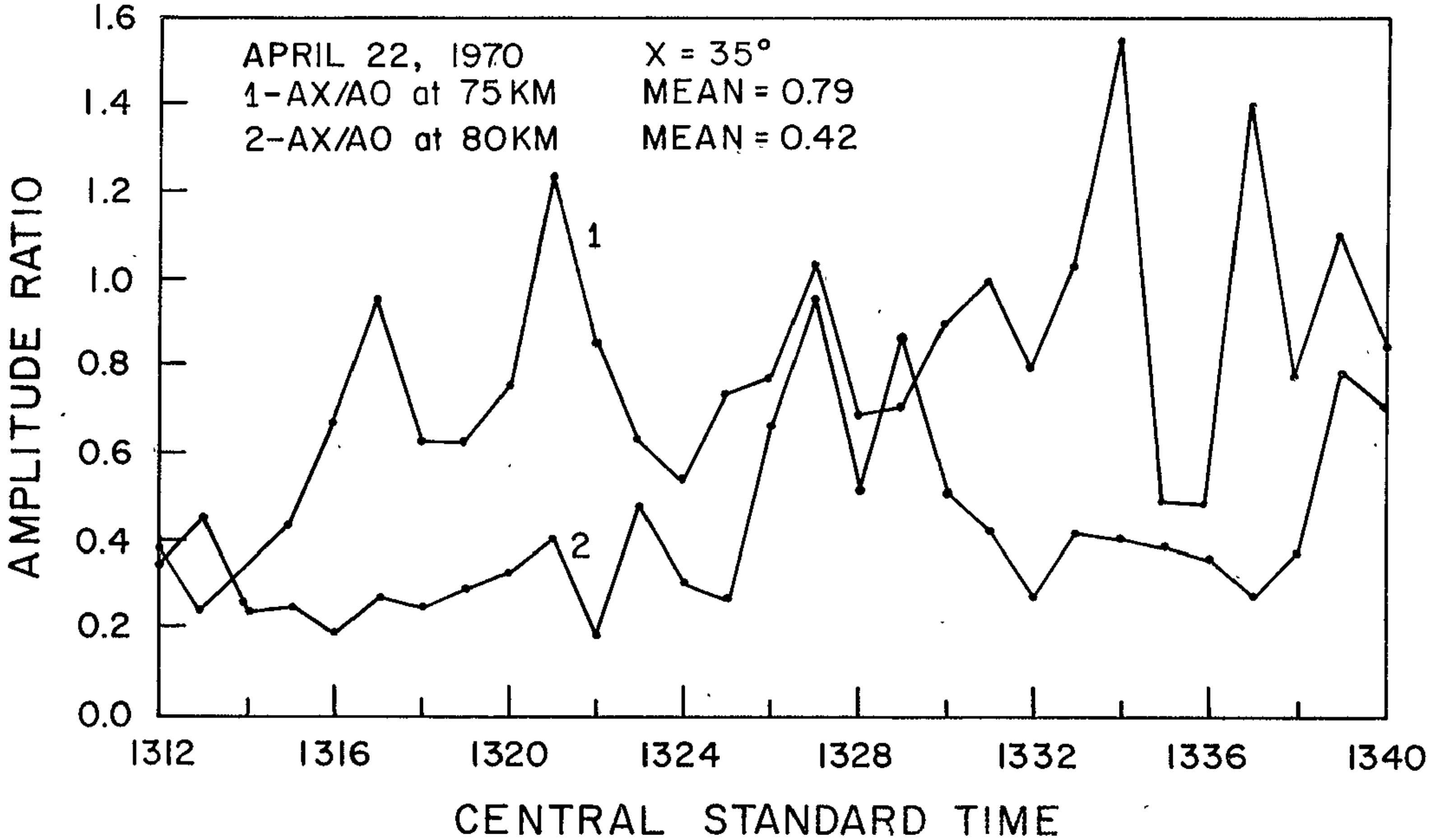


Figure 8.6 Amplitude ratios, April 22, 1970.

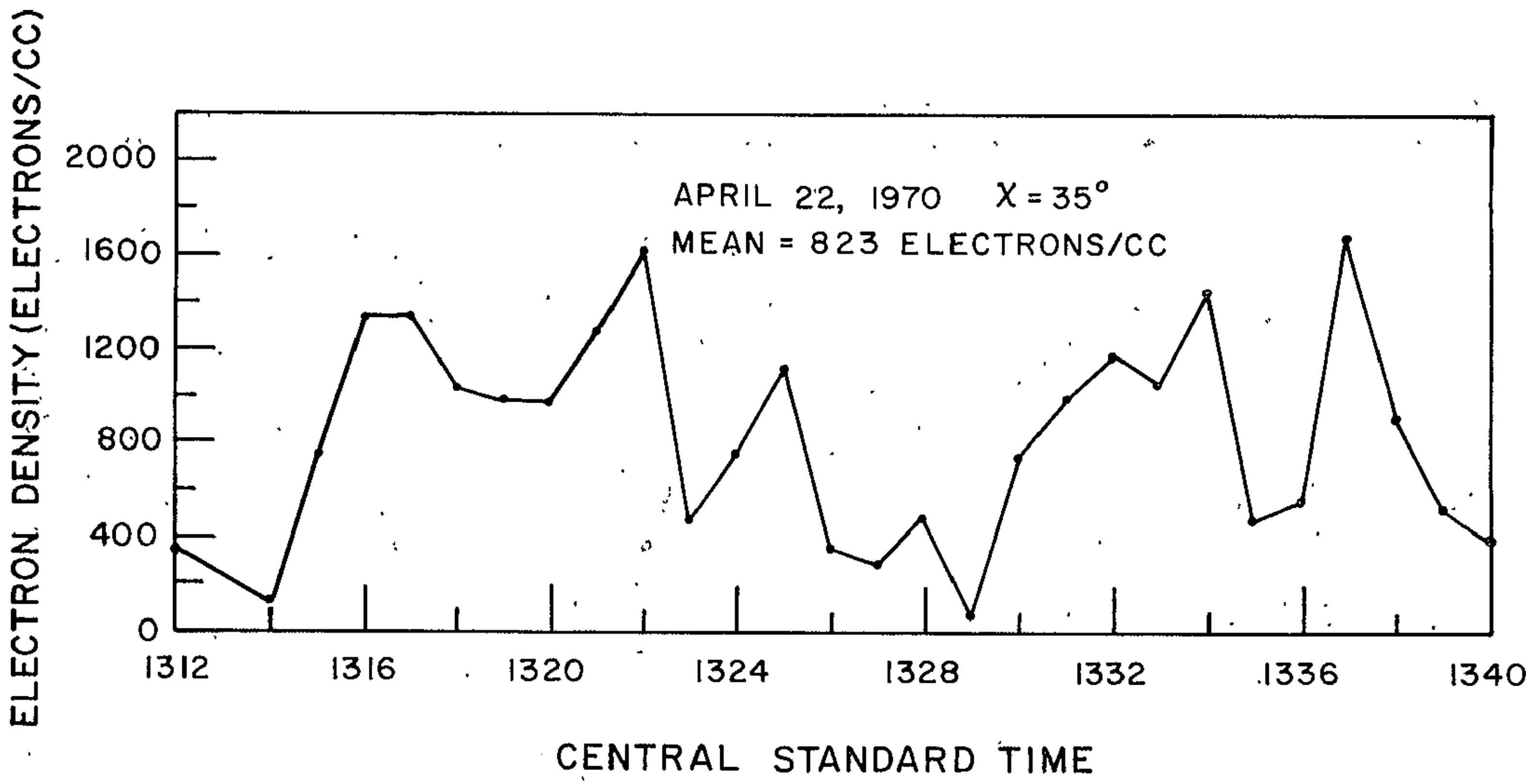


Figure 8.7 Electron density versus time, April 22, 1970.

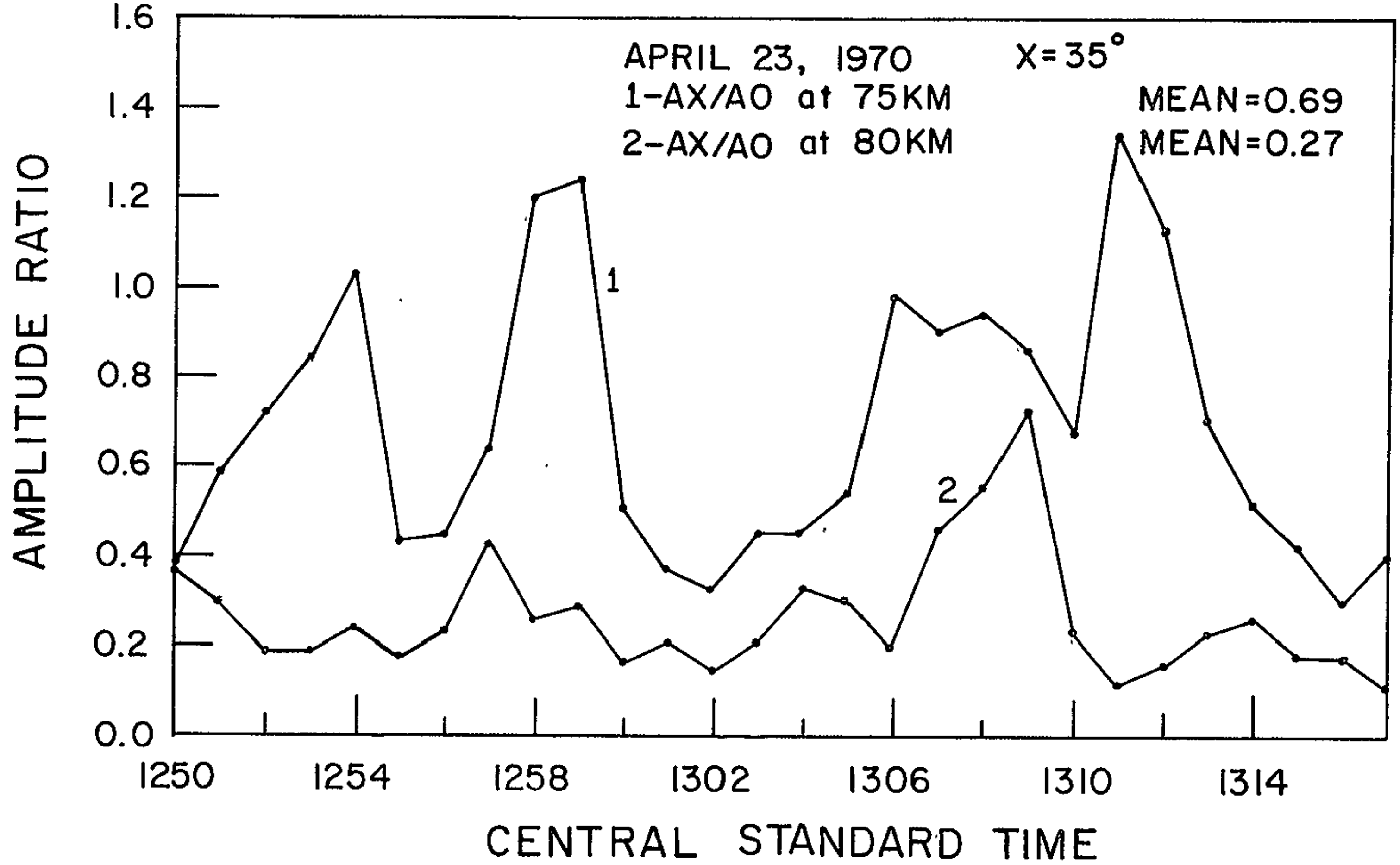


Figure 8.8 Amplitude ratios, April 23, 1970.

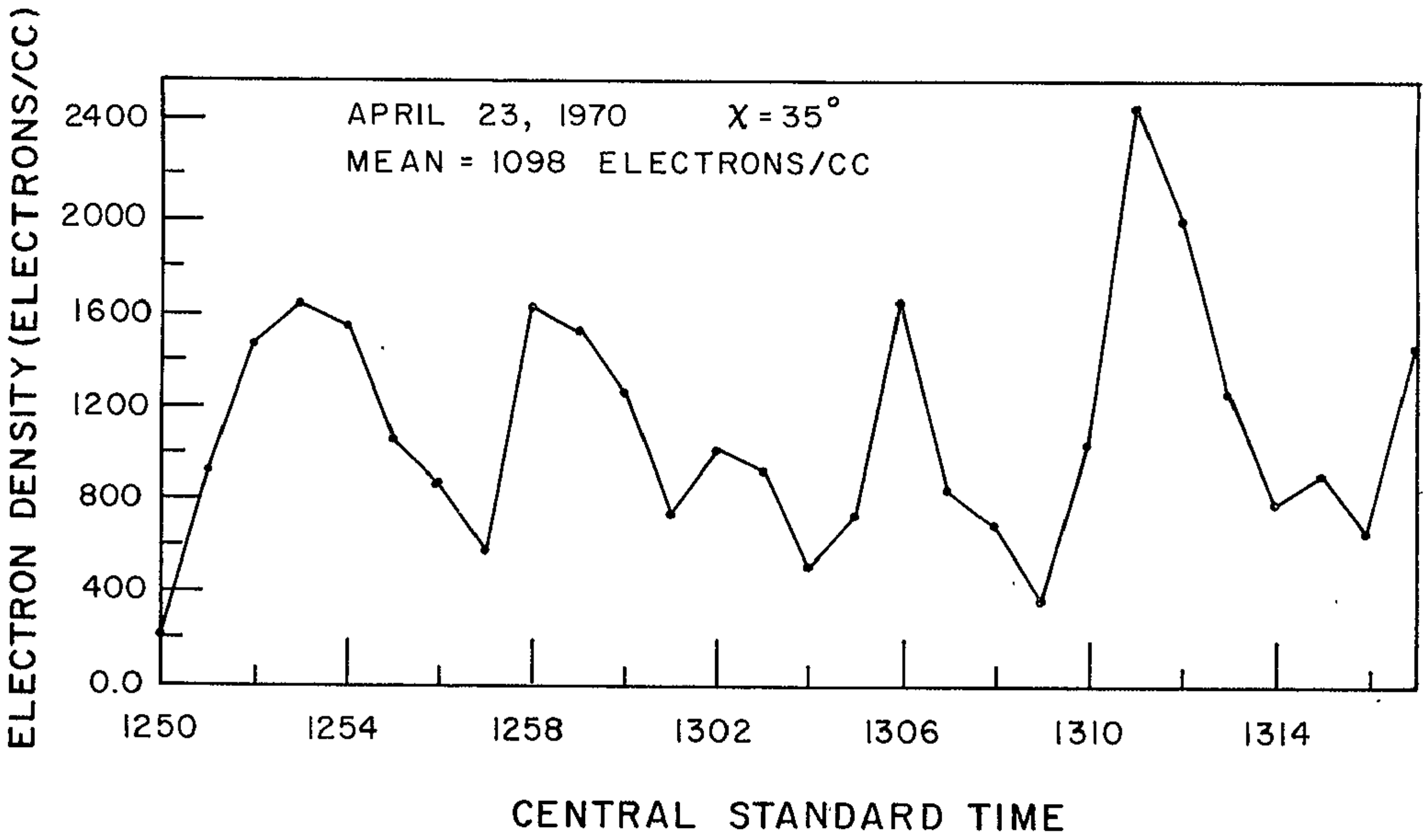


Figure 8.9 Electron density versus time, April 23, 1970.

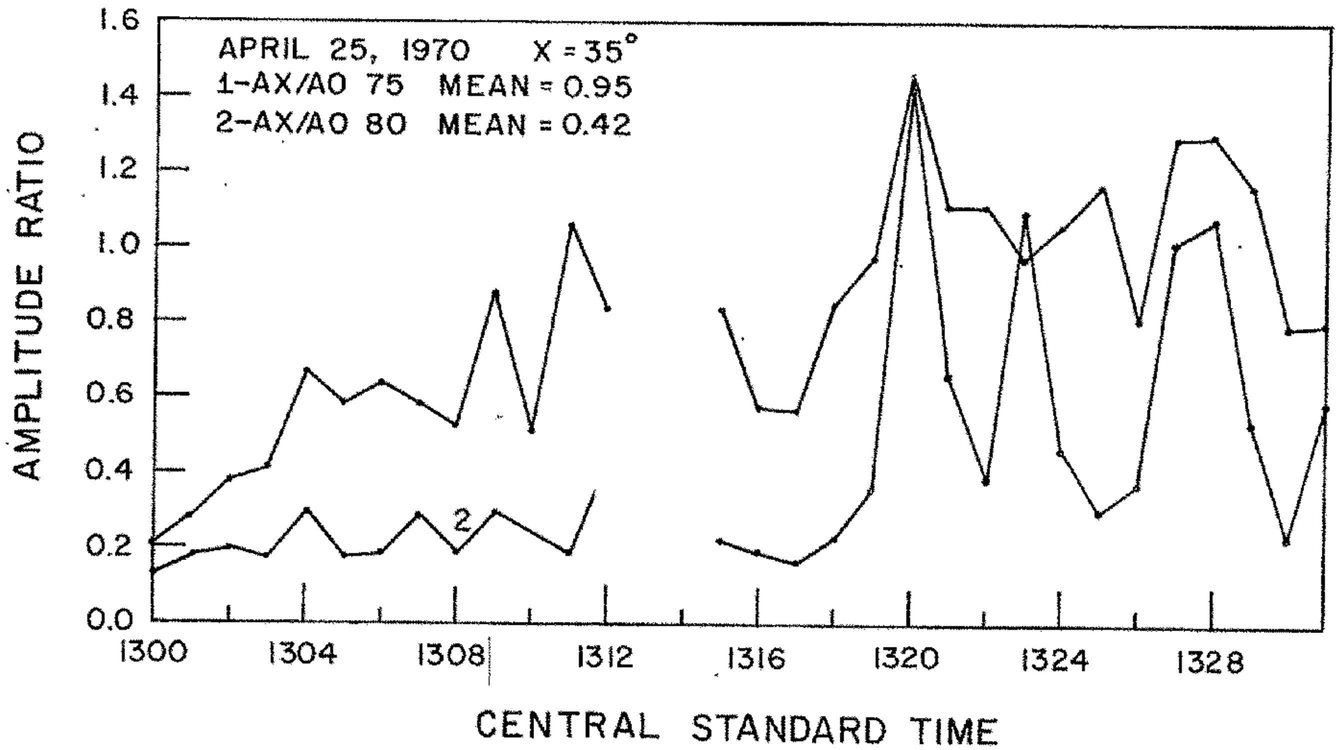


Figure 8.10 Amplitude ratios, April 25, 1970.

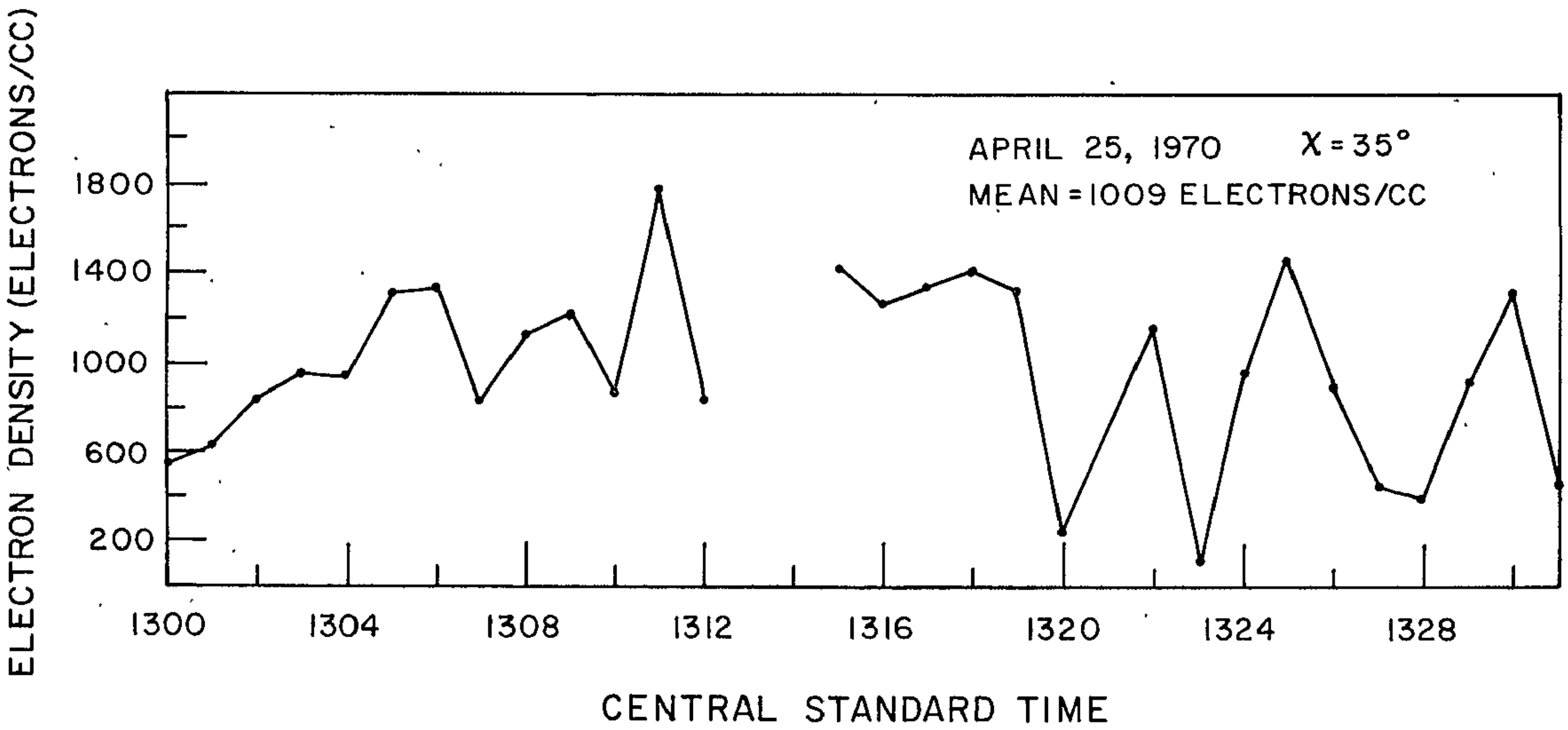
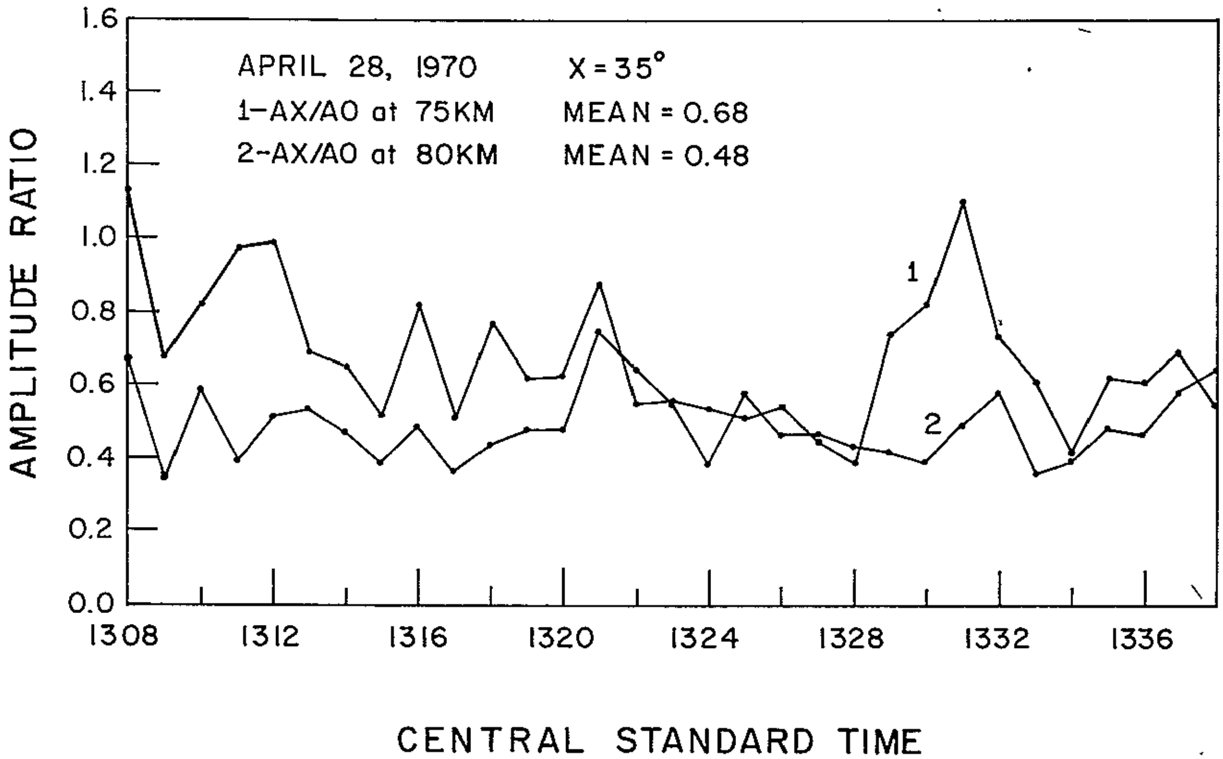


Figure 8.11 Electron density versus time, April 25, 1970.



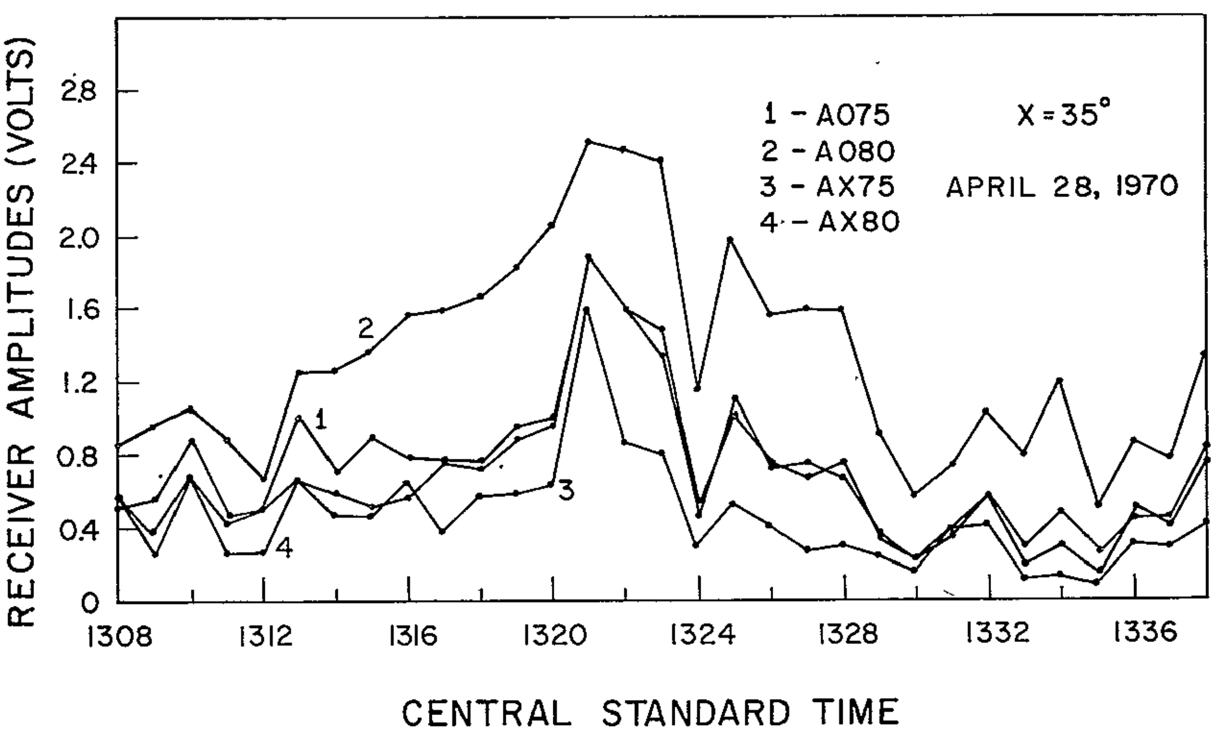


Figure 8.12 Receiver amplitudes and amplitude ratios, April 28, 1970.

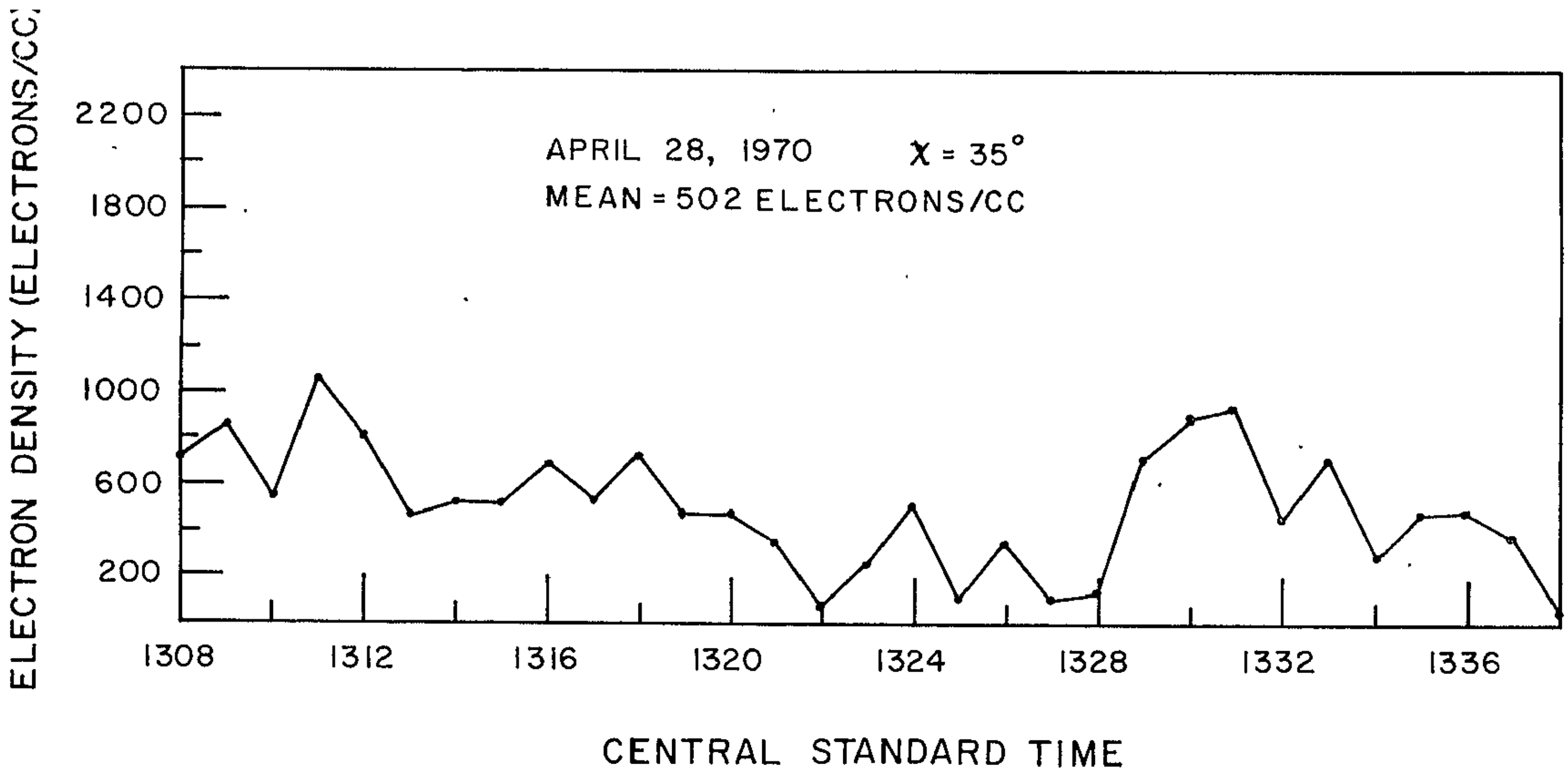
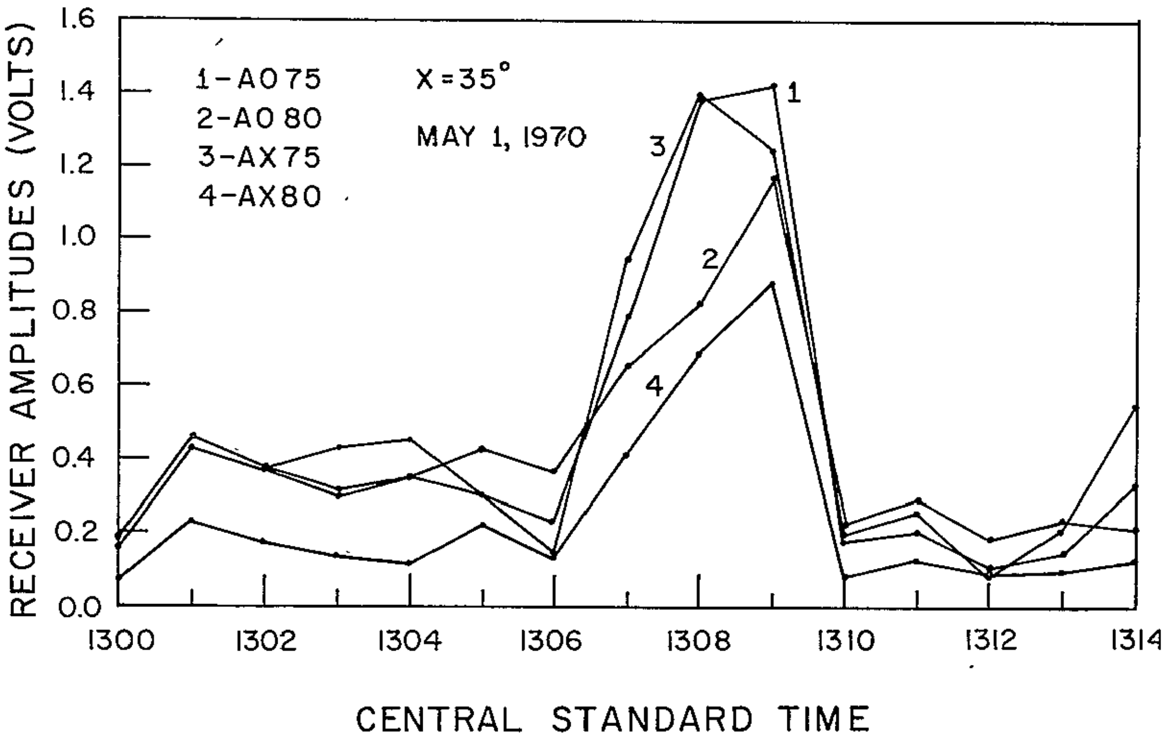


Figure 8.13 Electron density versus time, April 28, 1970.



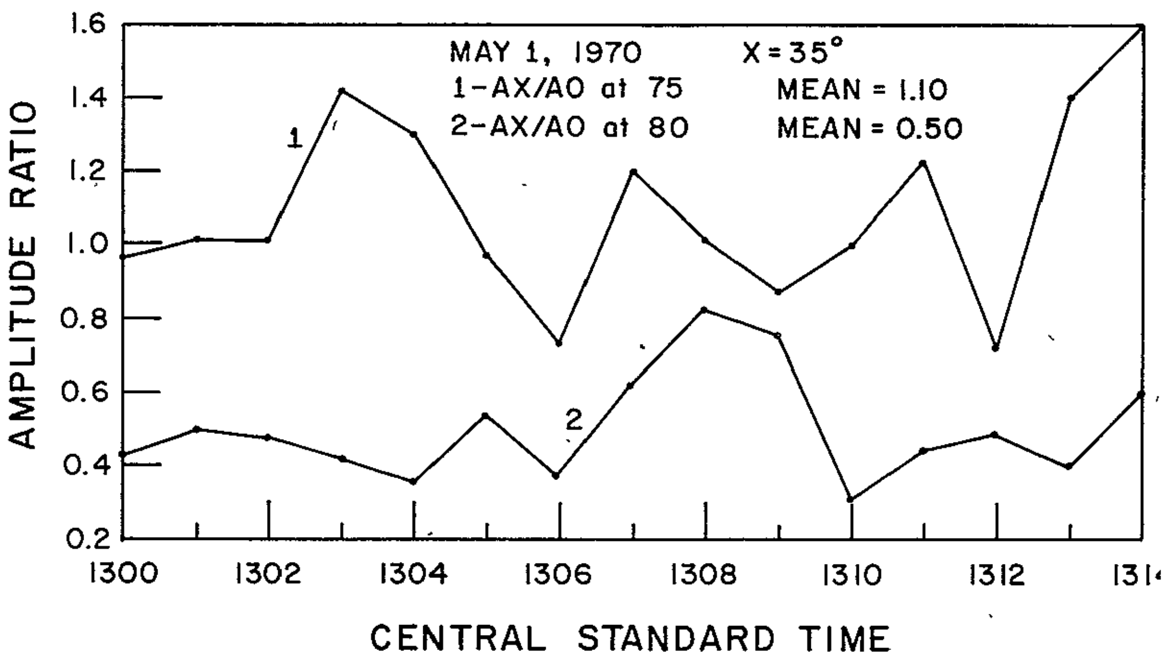


Figure 8.14 Receiver amplitude and amplitude ratios, May 1, 1970.

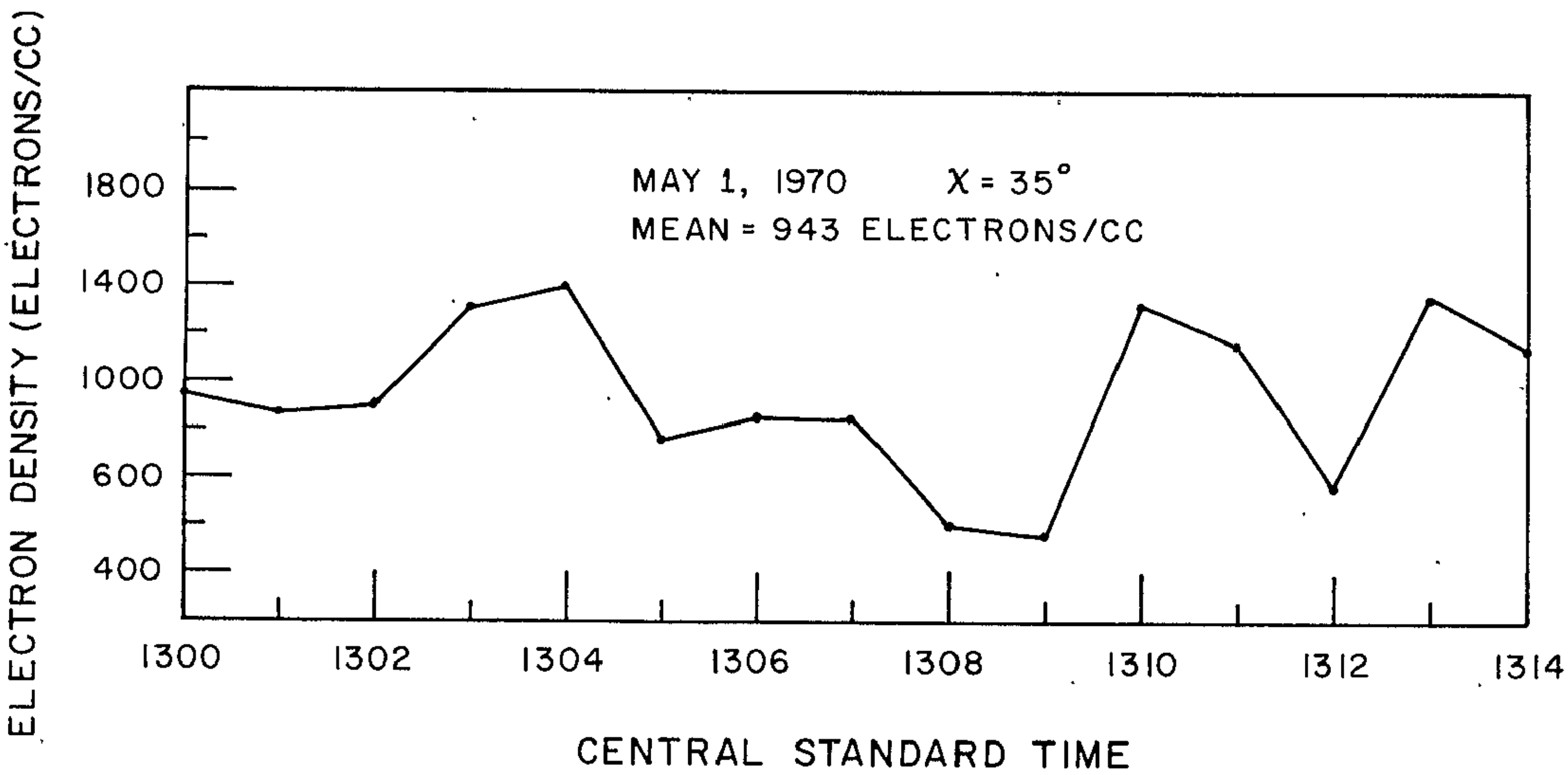


Figure 8.15 Electron density versus time, May 1, 1970.

9. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

Shown in Figure 9.1 are electron-density profiles generated from data obtained from seasonal rocket launches at Wallops Island, Virginia (Mechtly and Smith, 1968). For the month of April, the electron density measured at 77.5 km is approximately 320 electrons/cc. However, the solar zenith angle at launch was 60°. Data obtained from the SOMED instrumentation for April at a solar zenith angle of 35° shows an electron density in the range 900 to 1000 electrons/cc, a value which is certainly reasonable considering the increase at this altitude of electron density with solar zenith angle.

In a paper published by Belrose and Burke (Belrose and Burke, 1964) a plot of amplitude ratio, AX/AO, versus height is given. The curve was obtained by the experimenters using a transmitting frequency of 2.66 MHz on a quiet day, May 1, 1961. The value of AX/AO which they obtained at 75 and 80 km respectively was 0.85 and 0.33. Similar results have been obtained using the SOMED facility. For example, daily averages of AX/AO at 75 km have been 0.69 on April 21, 0.79 on April 22, 0.69 on April 23, 0.95 on April 25, 0.68 on April 28, and 1.10 on May 1. Daily averages of AX/AO at 80 km have been 0.30 on April 21, 0.42 on April 23, 0.27 on April 23, 0.42 on April 25, 0.49 on April 28, and 0.5 on May 1. All these values are certainly reasonable in comparison with the results of Belrose and Burke.

Even though the results obtained seem quite reasonable several suggestions can be made which might improve the reliability of the data. One suggestion would be to use a weighting function which would put more emphasis on data which are obtained during intervals when AO75, AO80, AX75 and AX80 are large. This would minimize contamination of the results by data recorded when partial reflections fade in the 75 to 80 km region.

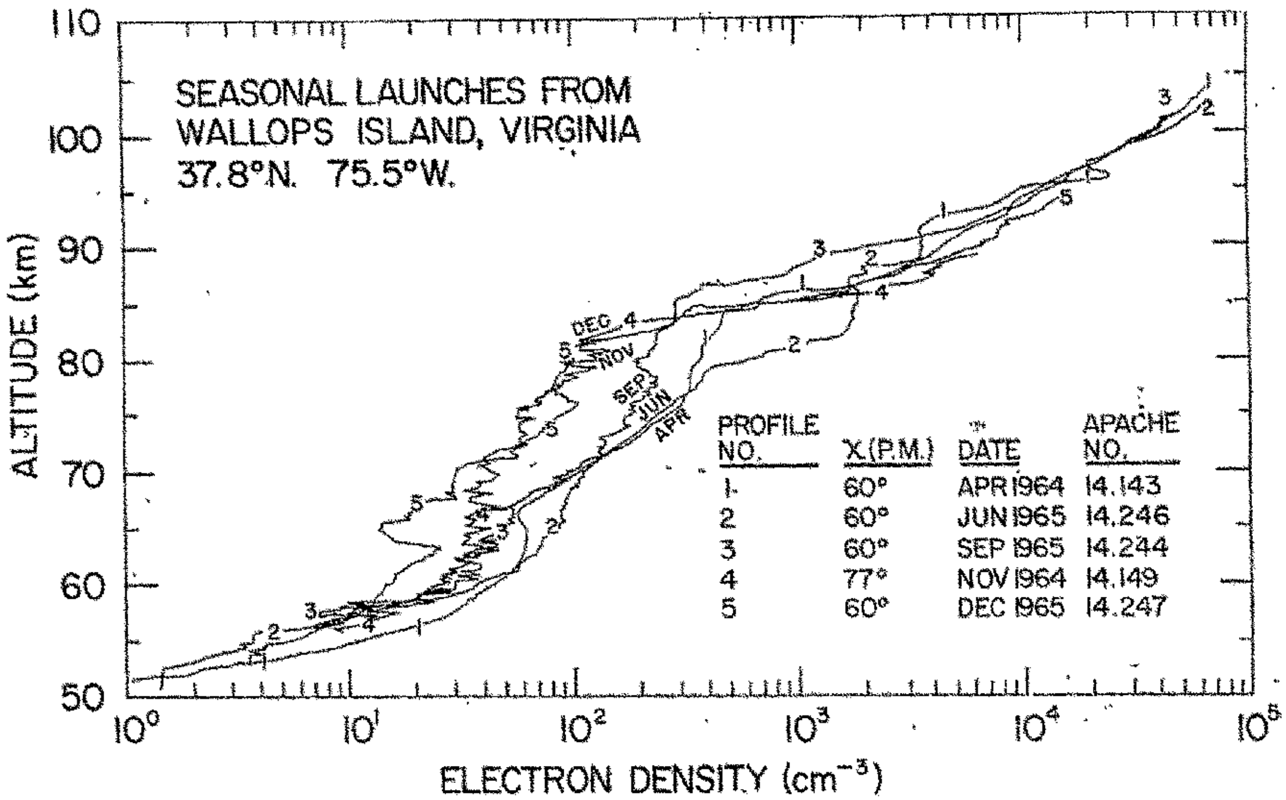


Figure 9.1 Electron-density profiles

Another suggestion would be to increase the averaging interval from 1 minute to 3 minutes. In this case, following the procedure of Chapter 8, we have

$$\sigma_y^2 = \frac{\sigma_X^2}{N} = \frac{0.75}{90} = 0.0083$$

Applying Tchebychev's inequality we have

$$P\{|y - M_X| \geq k(0.091)\} \leq \frac{1}{k^2} .$$

Letting k = 3.0 implies

$$P\{|y - M_X| \geq 0.27\} \leq 0.11 .$$

In other words, y is within 0.27 of the mean of X with a probability greater than or equal to 0.89. For our case y represents the averaged amplitude. Thus, the averaged amplitude would be within 0.27 volts of the mean of the amplitude distribution with a probability greater than or equal to 0.89. This is a significant improvement over the probability bounds for the one minute averaging interval.

A calculation similar to the above can be done for the 30 minute averaging interval used to obtain the means presented on the graphs of Chapter 8. For that case, N = 900. The result is

$$P\{|y - M_X| \geq 0.084\} \leq 0.11$$

This shows that the 30 minute average certainly attains the mean of the amplitude distribution.

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APPENDIX

List of Symbols Used In Computer Program

KK - paper tape character values in BCD

X - decoded integer paper tape characters

LM - a counting variable used to adjust paper tape reading procedure after first 24 characters

LC - character which controls the point at which the paper tape reader stops

NMIN - index of first number to be read off paper tape by paper tape reader

NMAX - index of last number to be read off paper tape by paper tape reader

L1 - a control variable used in decoding process

L2 - a control variable used in decoding process

L28 - a variable used in the character checking procedure to determine error type

LAVG - a variable which determines the basic averaging interval

AO75 - a variable used to obtain the average amplitude of the ordinary mode at 75 km

AO80 - a variable used to obtain the average amplitude of the ordinary mode at 80 km

AX75 - a variable used to obtain the average amplitude of the extraordinary mode at 75 km

AX80 - a variable used to obtain the average amplitude of the extraordinary mode at 80 km

RAT75 - amplitude ratio at 75 km

RAT80 - amplitude ratio at 80 km

BBB - a conversion variable necessary for use of electron density subroutine

EV - electron density

AOL - cumulative average value of amplitude of ordinary mode at 75 km

AOH - cumulative average value of amplitude of ordinary mode at 80 km

AXL - cumulative average value of amplitude of extraordinary mode at 75 km

AXH - cumulative average value of amplitude of extraordinary mode at 80 km

Z75 - cumulative average value of amplitude ratio at 75 km

Z80 - cumulative average value of amplitude ratio at 80 km

RAT075 - a variable used to obtain the cumulative average amplitude ratio at 75 km

RAT080 - a variable used to obtain the cumulative average amplitude ratio at 80 km

ZELEC - a variable used to obtain the cumulative average electron density

ZELECT - cumulative average value of electron density

L22 - a variable used to determine the number of samples over which the cumulative average was made

List of Symbols Used in Function ELDEN

GNU = collision frequency = ν_m

O = $(\omega+\omega_L)/\nu_m$

X = $(\omega-\omega_L)/\nu_m$

CTO = $\mathcal{C}_{3/2}[(\omega+\omega_L)/\nu_m]$

CTX = $\mathcal{C}_{3/2}[(\omega-\omega_L)/\nu_m]$

CFO = $\mathcal{C}_{5/2}[(\omega+\omega_L)/\nu_m]$

CFX = $\mathcal{C}_{5/2}[(\omega-\omega_L)/\nu_m]$

RX = R_x

RO = R_o

RXBYRO = R_x/R_o

AXBYAO = A_x/A_o

RATIO = $(A_x/A_o)/(R_x/R_o)$

DELTAH = Δh

AVGNU = ν_m at the center of Δh = $(\nu_m)_{\Delta h}$

FO = $(5e^2/m\varepsilon_o)\mathcal{C}_{5/2}[(\omega+\omega_L)/(\nu_m)_{\Delta h}]/4c(\nu_m)_{\Delta h}$

FX = $5(e^2/m\varepsilon_o)\mathcal{C}_{5/2}[(\omega-\omega_L)/(\nu_m)_{\Delta h}]/4c(\nu_m)_{\Delta h}$

FD = FX - FO

ELDEN = Electron Density = $\dfrac{\ln\{[(A_x/A_o)/(R_x/R_o)]_{h_1}/[(A_x/A_o)/(R_x/R_o)]_{h_2}\}}{2\Delta h FD}$

The Approximate Equations for the $\mathcal{C}$ Integrals Used in Function ELDEN

$$\mathcal{C}_{3/2}(X) = (X^4+a_3X^3+a_2X^2+a_1X+a_o)/(X^6+b_5X^5+b_4X^4+b_3X^3+b_2X^2+b_1X+b_o)$$

where

$a_o = 2.3983474 \times 10^{-2}$

$a_1 = 1.1287513 \times 10$

$a_2 = 1.1394160 \times 10^2$

$a_3 = 2.4653115 \times 10$

$b_o = 1.8064128 \times 10^{-2}$

$b_1 = 9.3877372$

$b_2 = 1.4921254 \times 10^2$

$b_3 = 2.8958085 \times 10^2$

$b_4 = 1.2049512 \times 10^2$

$b_5 = 2.4656819 \times 10$

$$\mathcal{C}_{5/2}(X) = (X^3+a_2X^2+a_1X+a_o)/(X^5+b_4X^4+b_3X^3+b_2X^2+b_1X+b_o)$$

where

$a_o = 1.1630641$

$a_1 = 1.6901002 \times 10$

$a_2 = 6.6945939$

$b_o = 4.3605732$

$b_1 = 6.4093464 \times 10$

$b_2 = 6.8920505 \times 10$

$b_3 = 3.5355257 \times 10$

$b_4 = 6.6314497$

```
      SINGLE
      DIMENSION KK(24),X(24),RAT75(720),RAT80(720),BEB(2)
      CALL SET (112,1)
C     TAPE IS NOW SET OVER FIRST POSITIVE SIGN
      LM=0
      RAT075=0.0
      RAT080=0.0
      L22=0
      ZELEC=0.0
      DO 75 K=1,720
      LAVG=30
C     LAVG=30 MEANS AVG INTERVAL OF ONE MINUTE
      A075=0
      A080=0
      AX75=0
      AX80=0
      DO 60 L=1,LAVG
      LM=LM+1
      LC =0
      NMAX=24
      L28=0
      IF(LM-1) 109,110,111
  109 CONTINUE
  110 KK(1)=112
      NMIN=2
      GO TO 150
  111 NMIN=1
  150 CONTINUE
      CALL IREC(KK,0,J,NMIN,NMAX)
      PRINT 115,(KK(J),J=1,24)
  115 FORMAT(4X,12F6.1)
      IF(KK(1)-LC)291,22,291
  291 IF(L28-1)13,280,13
  280 NMIN=1
      NMAX=24
      L28=0
      GO TO 150
C     24 CHARACTERS ARE READ IN AS A GROUP
   13 IF(KK(6)-128)200,201,200
  201 IF(KK(12)-128)202,203,202
  203 IF(KK(18)-128)204,214,204
  214 IF(KK(24)-128)206,205,206
  200 IF(KK(6)-32)209,207,209
  209 IF(KK(11)-128)208,210,208
  210 IF(KK(17)-128)208,211,208
  211 IF(KK(23)-128)208,212,208
  207 IF(KK(12)-128)208,213,208
  213 IF(KK(18)-128)208,215,208
  215 IF(KK(24)-128)208,216,208
  202 IF(KK(12)-32)217,218,217
  217 IF(KK(17)-128)208,219,208
  219 IF(KK(23)-128)208,212,208
  218 IF(KK(18)-128)208,220,208
  220 IF(KK(24)-128)208,216,208
  204 IF(KK(18)-32)221,222,221
  221 IF(KK(23)-128)208,212,208
  222 IF(KK(24)-128)208,216,208
  206 IF(KK(24)-32)205,216,205
  212 NMIN=2
      NMAX=24
      KK(1)=112
      GO TO 150
  216 L28=1
      NMIN=1
      NMAX=6
      GO TO 150
  205 CONTINUE
C     END LINE CHARACTERS CHECKED
C     DECODE DIGIT KK
      L1=2
      DO 85 I=1,4
      L2=L1+3
      DO 90 J=L1,L2
      IF(KK(J)-32) 1,19,2
    2 GO TO 80
    1 IF(KK(J)-16) 3,4,12
    4 GO TO 81
    3 X(J)=KK(J)
      GO TO 90
   12 X(J)=KK(J)-16
      GO TO 90
   19 X(J)=0
   90 CONTINUE
      L1=L1+6
```

```
 85    CONTINUE
C      ALL KK ARE NOW DECODED INTO X
C      NOW COMPUTE A075,A080,AX75,AX80
       A075=A075+10.0*X(2)+X(3)+0.1*X(4)+0.01*X(5)
       A080=A080+10.0*X(8)+X(9)+0.1*X(10)+0.01*X(11)
       AX75=AX75+10.0*X(14)+X(15)+0.1*X(16)+0.01*X(17)
       AX80=AX80+10.0*X(20)+X(21)+0.1*X(22)+0.01*X(23)
 60    CONTINUE
       AOL=A075/30.0
       AOH=A080/30.0
       AXL=AX75/30.0
       AXH=AX80/30.0
       RAT75(K)=AX75/A075
       RAT80(K)=AX80/A080
       PRINT 26, RAT75(K),RAT80(K),K
 26    FORMAT (4X,23HAX AT 75 TO AO AT 75 IS,F16.8,4X,23HAX AT 80 TO AO A
      1T 80 IS,F16.8,8X,11HTIME IS T +,I4,1X,7HMINUTES)
       PRINT 89,AOL,AOH,AXL,AXH
 89    FORMAT(4X,6HA075 =,F16.8,4X,6HA080 =,F16.8,4X,6HAX75 =,F16.8,4X,6H
      1AX80 =,F16.8)
       BBB(1)=RAT75(K)
       BBB(2)=RAT80(K)
       EN=ELDEN(BBB)
       IF(EN)77,77,76
 76    L22=L22+1
       ZELEC=ZELEC+EN
       RAT075=RAT75(K)+RAT075
       RAT080=RAT80(K)+RAT080
 77    CONTINUE
       PRINT 66,EN
 66    FORMAT (4X,17HELECTRON DENSITY=,1PE12.3,7H PER M3)
 75    CONTINUE
 208   PRINT 300,K
 300   FORMAT(4X,49HERROR, TWO MISPUNCHES IN 24 CHARACTER GROUP AT K=,I10
      1)
       GO TO 22
 80    PRINT 45,K
 45    FORMAT (4X,24HERROR, DIGIT GT 32 AT K=,I10)
       GO TO 22
 81    PRINT 46,K
 46    FORMAT (4X,21HERROR,DIGIT =16 AT K=,I10)
 22    Z=L22
       Z75=RAT075/Z
       Z80=RAT080/Z
       ZELECT=ZELEC/Z
       PRINT 53,Z75,Z80,ZELECT
 53    FORMAT(4X,20HAVERAGE AX75/A075 IS,F16.8,4X,20HAVERAGE AX80/A080 IS
      1,F16.8,4X,23HAVERAGE ELECT DENSITY =,1PE12.3)
       END
```

```
$      FORTRAN LDGO

C-----------------------------------------------------------------------C      SET
C        SUBROUTINE SET(NC,ND) -- STOPS PAPER TAPE AT A SPECIFIED       C      SET
C        CHARACTER, NC .  TAPE IS MOVED FORWARD IF ND .GE. 0 ,          C      SET
C         BACKWARDS IF ND .LT. 0 .                                      C      SET
C-----------------------------------------------------------------------C      SET
      SUBROUTINE SET(NC,ND)                                              1.0   SET
      SINGLE                                                             2.0   SET
      IF (ND) 5,10,10                                                    3.0   SET
    5 CALL PTBAC                                                         4.0   SET
      GO TO 15
   10 CALL PTFWD                                                         6.0   SET
   15 CALL PTRDS(K)                                                      7.0   SET
      IF (K-NC) 15,20,15                                                 8.0   SET
   20 CALL PTSTP                                                         9.0   SET
      RETURN                                                             10.0  SET
      END                                                                11.0  SET
      FORTRAN LDGO

C-----------------------------------------------------------------------C     LREC
C        SUBROUTINE LREC(KK,LC,J,NMAX) -- PLACES CHARACTERS              C     LREC
C        FROM PAPER TAPE IN ARRAY KK .  TRANSMISSION STOPS AT CHAR LC   C     LREC
C        OR AFTER NMAX CHARS .  J IS RETURNED AS NUMBER OF CHARS READ . C     LREC
C-----------------------------------------------------------------------C     LREC
      SUBROUTINE LREC (KK,LC,J,NMIN,NMAX)
      SINGLE                                                             2.0 LREC
      DIMENSION KK(1)                                                    3.0 LREC
      CALL PTFWD                                                         4.0 LREC
      DO 5 J=NMIN,NMAX
      CALL PTRDS(KK(J))                                                  6.0 LREC
      IF (KK(J)-LC) 5,10,5                                               7.0 LREC
    5 CONTINUE                                                           8.0 LREC
   10 CALL PTSTP                                                         9.0 LREC
      RETURN                                                             10.0LREC
      END                                                                11.0LREC
```

```
$       FORTRAN LDGC

C       ELECTRON DENSITY, FUNCTION SUBROUTINE
        FUNCTION ELDEN(AAA)
        DIMENSION AXBYAO(3),RXBYRO(3),RX(3),RO(3),GNU(3),AAA(2)
C       APPROX INTEGRAL PARAMETERS
        A1=2.3983474E-2
        A2=1.1287513E+1
        A3=1.1394160E+2
        A4=2.4653115E+1
        B1=1.8064128E-2
        B2=9.3877372
        B3=1.4921254E+2
        B4=2.8958085E+2
        B5=1.2049512E+2
        B6=2.4656819E+1
        D1=1.1630641
        D2=1.6901002E+1
        D3=6.6945939
        E1=4.3605732
        E2=6.4093464E+1
        E3=6.8920505E+1
        E4=3.5355257E+1
        E5=6.6314497
        AXBYAO(1)=AAA(1)
        AXBYAO(2)=AAA(2)
        AXBYAO(3)=0
C       CALCULATE COLLISION FREQUENCIES AT 75 AND 80 KM
C       MEAN PRESSURES, SEASON AND LATITUDE, NEWTONS/SQ.M.
        GNU(1)=7.0E+5*2.304
        GNU(2)=7.0E+5*9.745E-1
        GNU(3)=(GNU(1)+GNU(2))/2
C       CALCULATE C INTEGRALS AT BOTH HEIGHTS AND FOR AVERAGE GNU
        DO 22 K=1,3
        O=(2.59614E+7)/(GNU(K))
        X=(7.3886E+6)/(GNU(K))
        CTN=O*(O*(O*(O+A1)+A2)+A3)+A4
        CTD=O*(O*(O*(O*(O*(O+B1)+B2)+B3)+B4)+B5)+B6
        CTO=CTN/CTD
        CTXN=X*(X*(X*(X+A1)+A2)+A3)+A4
        CTXD=X*(X*(X*(X*(X*(X+B1)+B2)+B3)+B4)+B5)+B6
        CTX=CTXN/CTXD
        CFO=(O*(O*(O+D1)+D2)+D3)/(O*(O*(O*(O*(O+E1)+E2)+E3)+E4)+E5)
        CFX=(X*(X*(X+D1)+D2)+D3)/(X*(X*(X*(X*(X+E1)+E2)+E3)+E4)+E5)
C       CALCULAATE RATIOS
        RX(K)=SQRT((X*CTX)**2+(2.5*CFX)**2)
        RO(K)=SQRT((O*CTO)**2+(2.5*CFO)**2)
        RXBYRO(K)=RX(K)/RO(K)
   22   RATIO(K)=AXBYAO(K)/RXBYRO(K)
C       CALCULATE FD FROM FINAL VALUES OF DO LOOP
        FO=(5.*3.1824E+3*CFO)/(4.*3.E+8*GNU(3))
        FX=(5.*3.1824E+3*CFX)/(4.*3.E+8*GNU(3))
        FD=FX-FO
C       ELECTRON DENSITY AT 77.5 KM
        ELDEN= LOG(RATIO(1)/RATIO(2))/(2.*5.E+3*FD)
        RETURN
        END

$       DATA  I+
```